

Lesson 11-4

Example 1 Find the Sum of the First n Terms

Find S_n for each geometric series described.

a. $a_1 = 3, a_6 = -96, r = -2$

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{Sum formula}$$

$$S_6 = \frac{3[1 - (-2)^6]}{1 - (-2)} \quad a_1 = 3, r = -2, n = 6$$

$$S_7 = -63 \quad \text{Use a calculator.}$$

The sum of the first 6 terms of the series is -63 .

b. $a_1 = 8, r = \frac{2}{5}, n = 5$

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{Sum formula}$$

$$S_5 = \frac{8 \left[1 - \left(\frac{2}{5} \right)^5 \right]}{1 - \frac{2}{5}} \quad a_1 = 8, r = \frac{2}{5}, n = 5$$

$$= \frac{8 \left(1 - \frac{32}{3125} \right)}{\frac{3}{5}} \quad \left(\frac{2}{5} \right)^5 = \frac{32}{3125}$$

$$= \frac{8248}{625} \text{ or } 13.1968$$

Example 2 Evaluate a Sum Written in Sigma Notation

Evaluate $\sum_{n=1}^{10} 5 \left(\frac{1}{2} \right)^{n-1}$.

To find the sum, you can find the terms by replacing n with 1 through 10. Then you can add the terms. However, there are 10 terms, so it will be quicker to use the sum of a geometric series formula.

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{Sum formula}$$

$$S_{10} = \frac{5 \left[1 - \left(\frac{1}{2} \right)^{10} \right]}{1 - \frac{1}{2}} \quad n = 10, a_1 = 5, r = \frac{1}{2}$$

$$S_{10} = \frac{\left(\frac{5115}{1024}\right)}{\left(\frac{1}{2}\right)} \quad \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

$$S_{10} = \frac{5115}{512} \quad \text{Simplify.}$$

The sum of the series is $\frac{5115}{512}$.

Example 3 Use the Alternate Formula for a Sum

Find the sum of a geometric series for which $a_1 = 7776$, $a_n = \frac{1}{36}$, and $r = \frac{1}{6}$.

Since you do not know the value of n , use the formula $S_n = \frac{a_1 - a_n r}{1 - r}$.

$$S_n = \frac{a_1 - a_n r}{1 - r} \quad \text{Alternate sum formula}$$

$$= \frac{7776 - \left(\frac{1}{36}\right)\left(\frac{1}{6}\right)}{1 - \frac{1}{6}}$$

$$a_1 = 7776, a_n = \frac{1}{36}, r = \frac{1}{6}$$

$$= \frac{1679615}{5}$$

$$= \frac{216}{6}$$

Simplify.

$$= \frac{335923}{36} \text{ or } 9331 \frac{7}{36}$$

Simplify.

The sum of the series is $9331 \frac{7}{36}$.

Example 4 Find the First Term of a Series

Find a_1 in a geometric series for which $S_5 = -28.1875$ and $r = -2.5$.

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \quad \text{Sum formula}$$

$$S_5 = \frac{a_1[1 - (-2.5)^5]}{1 - (-2.5)} \quad r = -2.5$$

$$-28.1875 = \frac{98.65625a_1}{3.5} \quad S_5 = -28.1875$$

$$-28.1875 = 28.1875a_1 \quad \text{Simplify.}$$

$$-1 = a_1 \quad \text{Divide each side by 28.1875.}$$

The first term of the series is -1 .