

## Lesson 11–6

### Example 1 Use a Recursive Formula

Find the first five terms of the sequence in which  $a_1 = -5$  and  $a_{n+1} = -2a_n - n + 5$ ,  $n \geq 1$ .

$$a_{n+1} = -2a_n - n + 5 \quad \text{Recursive formula}$$

$a_{1+1} = -2a_1 - (1) + 5$	$n = 1$		$a_{3+1} = -2a_3 - (3) + 5$	$n = 3$
$a_2 = -2(-5) - (1) + 5$ or 14	$a_1 = -5$		$a_4 = -2(-25) - (3) + 5$ or 52	$a_3 = -25$
$a_{2+1} = -2a_2 - (2) + 5$	$n = 2$		$a_{4+1} = -2a_4 - (4) + 5$	$n = 4$
$a_3 = -2(14) - (2) + 5$ or -25	$a_2 = 14$		$a_5 = -2(52) - (4) + 5$ or -103	$a_4 = 52$

The first five terms of the sequence are  $-5$ ,  $14$ ,  $-25$ ,  $52$ , and  $-103$ .

### Example 2 Find and Use a Recursive Formula

**INVESTING** Jodi starts a new job and is given a signing bonus of \$1000. She decides to place the \$1000 in a special savings account that earns 5.4% annual interest, compounded monthly on the last day of each month. She will then add \$100 to the account on the first day of each month.

- a. Write a recursive formula for the amount of money Jodi will have in her account on the first day of each month.

Let  $i_n$  represent the amount of money on the first day of each month. Since interest is added once per month, the monthly rate is  $5.4\% \div 12$ , which is  $0.45\%$  or  $0.0045$  in decimal form. The interest on her balance is added on the last day of the month, so her new balance is  $1.0045i_n$ . Then the next day she adds \$100 to her balance, so her balance is now  $i_{n+1} = 1.0045i_n + 100$ .

- b. Find the balance in Jodi's account at the beginning of the first five months of her savings plan. (At the beginning of the first month, her balance is \$1000.)

$$i_{n+1} = 1.0045i_n + 100 \quad \text{Recursive formula}$$

$i_{1+1} = 1.0045i_1 + 100$	$n = 1$
$i_2 = 1.0045(1000) + 100 = 1104.50$	$i_1 = 1000$

$i_{2+1} = 1.0045i_2 + 100$	$n = 2$
$i_3 = 1.0045(1104.5) + 100 \approx 1209.47$	$i_2 = 1104.50$

$i_{3+1} = 1.0045i_3 + 100$	$n = 3$
$i_4 = 1.0045(1209.47) + 100 \approx 1314.91$	$i_3 = 1209.47$

$i_{4+1} = 1.0045i_4 + 100$	$n = 4$
$i_5 = 1.0045(1314.91) + 100 \approx 1420.83$	$i_4 = 1314.91$

Her account balances for the first five months are \$1000, \$1104.50, \$1209.47, \$1314.91, and \$1420.83.

### Example 3 Iterate a Function

Find the first three iterates  $x_1, x_2, x_3$  of the function  $f(x) = x^2 - 1$  for an initial value of  $x_0 = -2$ .

To find the first iterate  $x_1$ , find the value of the function for  $x_0 = -2$ .

$$\begin{aligned}x_1 &= f(x_0) && \text{Iterate the function.} \\ &= f(-2) && x_0 = -2 \\ &= (-2)^2 - 1 \text{ or } 3 && \text{Simplify.}\end{aligned}$$

To find the second iterate  $x_2$ , substitute  $x_1$  for  $x$ .

$$\begin{aligned}x_2 &= f(x_1) && \text{Iterate the function.} \\ &= f(3) && x_1 = 3 \\ &= (3)^2 - 1 \text{ or } 8 && \text{Simplify.}\end{aligned}$$

Substitute  $x_2$  for  $x$  to find the third iterate.

$$\begin{aligned}x_3 &= f(x_2) && \text{Iterate the function.} \\ &= f(8) && x_2 = 8 \\ &= (8)^2 - 1 \text{ or } 63 && \text{Simplify.}\end{aligned}$$

Therefore,  $-2, 3, 8, 63$  is an example of a sequence generated using iteration.