

Lesson 12–8

Example 1 Binomial Theorem

Suppose you take a multiple-choice test with six questions for which each question has five answer choices. What is the probability of getting exactly three questions right?

There are two possible outcomes for each question: right or wrong. The probability of a right answer r is $\frac{1}{5}$ and the probability of a wrong answer w is $\frac{4}{5}$.

$$(r + w)^6 = r^6 + 6r^5w + 15r^4w^2 + 20r^3w^3 + 15r^2w^4 + 6rw^5 + w^6$$

The term $20r^3w^3$ represents 3 right and 3 wrong answers.

$$\begin{aligned} P(3 \text{ right, } 3 \text{ wrong}) &= 20r^3w^3 \\ &= 20\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^3 && r = \frac{1}{5}, w = \frac{4}{5} \\ &= \frac{256}{3125} \end{aligned}$$

The probability of getting exactly three questions right is $\frac{256}{3125}$ or about 8%.

Example 2 Binomial Experiment

QUALITY CONTROL A company makes fragile glass ornaments. At the end of the manufacturing process, 15% of the ornaments are defective. Five ornaments are selected at random during a particular shift.

a. What is the probability that exactly 3 ornaments selected are defective?

The probability of a defective ornament is $\frac{15}{100}$ or $\frac{3}{20}$. The probability that an ornament is not defective is $\frac{17}{20}$. There are $C(5, 3)$ ways to choose the defective ornaments.

$$\begin{aligned} P(3 \text{ defective ornaments}) &= C(5, 3)\left(\frac{3}{20}\right)^3\left(\frac{17}{20}\right)^2 && \text{If 3 ornaments are defective, 2 ornaments are not} \\ & && \text{defective.} \\ &= \frac{5 \cdot 4}{2}\left(\frac{3}{20}\right)^3\left(\frac{17}{20}\right)^2 && C(5, 3) = \frac{5!}{3! 2!} \\ &= \frac{7803}{320,000} && \text{Simplify.} \end{aligned}$$

The probability that exactly 3 ornaments out of the 5 selected are defective is $\frac{7803}{320,000}$ or about 0.024.

b. What is the probability that no more than 2 ornaments of the five chosen are defective?

No more than 2 defective means 0, 1, or 2 defective. Add the probabilities of 0 defective, 1 defective, and 2 defective.

$$\begin{aligned} P(\text{no more than 2 defective}) &= P(0 \text{ defective}) + P(1 \text{ defective}) + P(2 \text{ defective}) \\ &= C(5, 0) \left(\frac{3}{20}\right)^0 \left(\frac{17}{20}\right)^5 + C(5, 1) \left(\frac{3}{20}\right)^1 \left(\frac{17}{20}\right)^4 + C(5, 2) \left(\frac{3}{20}\right)^2 \left(\frac{17}{20}\right)^3 \\ &= \frac{1,557,421}{1,600,000} \quad \text{Simplify.} \end{aligned}$$

The probability that no more than 2 out of the five selected will be defective is $\frac{1,557,421}{1,600,000}$ or about 0.97.