Lesson 2–6

Example 1 Step Function

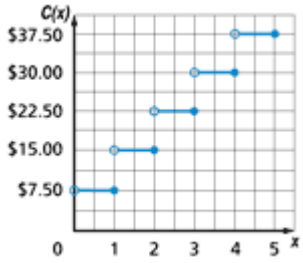
FITNESS A fitness center charges customers $7.50 per hour or any fraction thereof. Draw a graph that represents this situation.

Explore The total charge must be a multiple of 7.5, so the graph will be the graph of a step function.

Plan If the time spent at the center is greater than 0 hours, but less than or equal to 1 hour, then the charge will be $7.50. If the time is greater than 1 hour but less than or equal to 2 hours, then the labor cost is $15.00, and so on.

Solve Use the pattern of times and charges to make a table, where x is the number of hours spent at the center and $C(x)$ is the total charge. Then draw the graph.

x	$C(x)$
$0 < x \leq 1$	$7.50
$1 < x \leq 2$	$15.00
$2 < x \leq 3$	$22.50
$3 < x \leq 4$	$30.00
$4 < x \leq 5$	$37.50



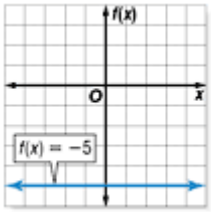
Examine Since the fitness center rounds any fraction of an hour up to the next whole number, each segment on the graph has a circle at the left endpoint and a dot at the right endpoint.

Example 2 Constant Function

Graph $f(x) = -5$.

For every value of x, $f(x) = -5$. The graph is a horizontal line.

$f(x) = -5$	
x	$f(x)$
–3	–5
–1.5	–5
0	–5
$\frac{3}{4}$	–5
2.7	–5



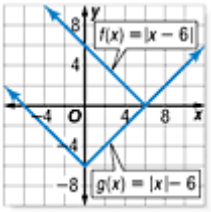
Example 3 Absolute Value Functions

Graph $f(x) = |x| - 6$ and $g(x) = |x - 6|$ on the same coordinate plane. Determine the similarities and differences in the two graphs.

Find several ordered pairs for each function.

x	$\|x\| - 6$
–2	–4
–1	–5
0	–6
1	–5
2	–4

x	$\|x - 6\|$
0	6
2	4
4	2
6	0
8	2

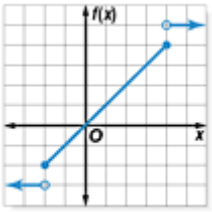


- Graph the points and connect them.
- The domain of each function is all real numbers.
- The range of $f(x) = |x| - 6$ is $\{y \mid y \geq -6\}$. The range of $g(x) = |x - 6|$ is $\{y \mid y \geq 0\}$.
- The graphs have the same shape, but different y-intercepts.
- The graph of $g(x) = |x - 6|$ is the graph of $f(x) = |x| - 6$ translated 6 units up and 6 units to the right.

Example 4 Piecewise Function

Graph $f(x) = \begin{cases} 5 \text{ if } x > 4 \\ x \text{ if } -2 \le x \le 4 \\ -3 \text{ if } x < -2 \end{cases}$. Identify the domain and range.

Step 1 Graph the constant function $f(x) = 5$ for $x > 4$. Since 4 does not satisfy this inequality, begin with an open circle at (4, 5).



Step 2 Graph the linear function $f(x) = x$ for $-2 \le x \le 4$. Since –2 and 4 both satisfy this inequality, begin with a closed circle at (–2, –2) and draw a line to the closed circle at (4, 4).

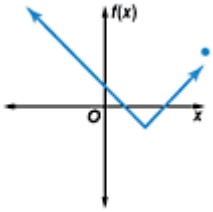
Step 3 Graph the constant function $f(x) = -3$ for $x < -2$. Since –2 does not satisfy this inequality, stop with an open circle at (–2, –3).

The function is defined for all values of x, so the domain is all real numbers. The values that are y-coordinates of points on the graph are 5, all real numbers greater than or equal to –2 but less than or equal to 4, and –3. The range is $\{y \mid y = 5 \text{ or } -2 \le y \le 4, \text{ or } y = -3\}$.

Example 5 Identify Functions

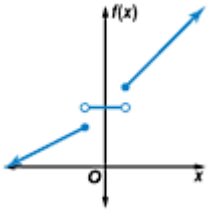
Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

a.



Since this graph is V-shaped, it represents an absolute value function.

b.



Since this graph consists of different rays and segments, it represents a piecewise function.