

Lesson 3-3

Example 1 Intersecting Regions

Solve each system of inequalities by graphing.

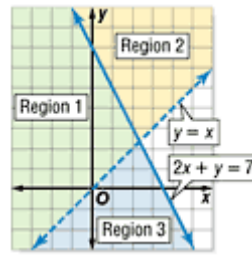
a. $y > x$

$$2x + y \leq 7$$

solution of $y > x \rightarrow$ Regions 1 and 2

solution of $2x + y \leq 7 \rightarrow$ Regions 1 and 3

The intersection of these regions is Region 1, which is the solution of the system of inequalities. Notice that the solution is a region containing an infinite number of ordered pairs.

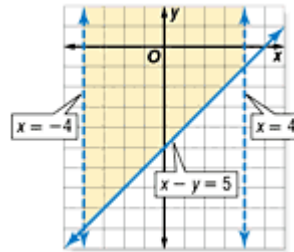


b. $x - y \leq 5$

$$|x| < 4$$

The inequality $|x| < 4$ can be written as $x < 4$ and $x > -4$.

Graph all of the inequalities on the same coordinate grid and shade the region or regions that are common to all.



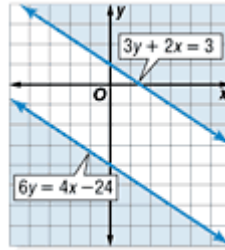
Example 2 Separate Regions

Solve the system of inequalities by graphing.

$$3y + 2x \geq 3$$

$$6y \leq -4x - 24$$

Graph both inequalities. The graphs do not overlap, so the solutions have no points in common. The solution set is \emptyset .



Example 3 Write and Use a System of Inequalities

The U.S. Postal Service allows packages up to 70 pounds with a combined length and girth not over 108 inches to be mailed under the classification of Priority Mail. Write and graph a system of inequalities that represents the range of weights and combined length and girth measures for Priority Mail. Source: The World Almanac

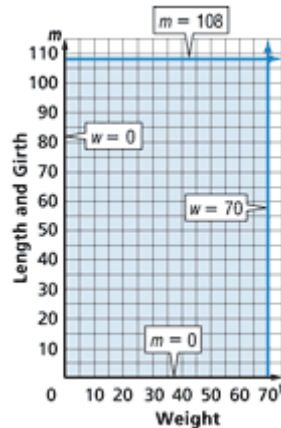
Let w represent the weight of packages in pounds. The acceptable weights are 0 to 70 pounds. We can write this information as two inequalities.

$$0 \leq w \text{ and } w \leq 70$$

Let m represent the combined length and girth of a package. The acceptable measures can also be written as two inequalities.

$$0 \leq m \text{ and } m \leq 108$$

Graph all of the inequalities. Any ordered pair in the intersection of the graphs is a solution of the system.

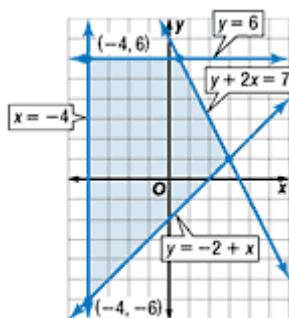


Example 4 Find Vertices

Find the coordinates of the vertices of the figure formed by $y \leq 6$, $x \geq -4$, $y \geq x - 2$, and $y + 2x \leq 7$.

Graph each inequality. The intersection of the graphs forms a quadrilateral.

The coordinates $(-4, 6)$ and $(-4, -6)$ can be determined from the graph. To find the coordinates of the other two vertices, you need to solve two systems.



System 1

$$\begin{aligned}y &= 6 \\y + 2x &= 7\end{aligned}$$

Substitute 6 for y into the second equation.

$$\begin{aligned}y + 2x &= 7 && \text{Second equation} \\6 + 2x &= 7 && \text{Replace } y \text{ with } 6. \\2x &= 1 && \text{Subtract } 6 \text{ from each side.} \\x &= \frac{1}{2} && \text{Divide each side by } 2.\end{aligned}$$

The third vertex is $\left(\frac{1}{2}, 6\right)$.

System 2

$$\begin{aligned}y + 2x &= 7 \\y &= -2 + x\end{aligned}$$

Rewrite the equations and subtract to eliminate y .

$$\begin{aligned}y + 2x &= 7 \\(-) y - x &= -2 \\ \hline 3x &= 9 && \text{Subtract the equations.} \\x &= 3 && \text{Divide each side by } 3.\end{aligned}$$

Now find y by substituting 3 for x in the first equation.

$$\begin{aligned}y + 2x &= 7 && \text{First equation} \\y + 2(3) &= 7 && \text{Substitute } 3 \text{ for } x. \\y + 6 &= 7 && \text{Multiply.} \\y &= 1 && \text{Subtract } 6 \text{ from each side.}\end{aligned}$$

The fourth vertex is $(3, 1)$.

The vertices of the quadrilateral are $(-4, 6)$, $(-4, -6)$, $\left(\frac{1}{2}, 6\right)$, and $(3, 1)$.