Lesson 3–5

Example 1 One Solution

Solve the system of equations.

$\mathbf{a + 2b - 4c = 8}$
$\mathbf{2a - b + c = -8}$
$\mathbf{-a - 3b + 2c = -9}$

Step 1 Use elimination to make a system of two equations in two variables.

$a + 2b - 4c = 8$	Multiply by 2.	$2a + 4b - 8c = 16$	First equation
$2a - b + c = -8$		$(-)\ 2a - b + c = -8$	Second equation
		$5b - 9c = 24$	Subtract to eliminate *a*.

$a + 2b - 4c = 8$	First equation
$(+)\ -a - 3b + 2c = -9$	Third equation
$-b - 2c = -1$	Add to eliminate *a*.

Notice that the *a* terms in each equation have been eliminated. The result is two equations with the same two variables *b* and *c*.

Step 2 Solve the system of two equations.

$5b - 9c = 24$		$5b - 9c = 24$	
$-b - 2c = -1$	Multiply by 5.	$(+)\ -5b - 10c = -5$	
		$-19c = 19$	Add to eliminate *b*.
		$c = -1$	Divide each side by –19.

Substitute –1 for *c* in one of the equations with two variables and solve for *b*.

$-b - 2c = -1$	Equation with two variables
$-b - 2(-1) = -1$	Replace *c* with –1.
$-b + 2 = -1$	Multiply.
$b = 3$	Simplify.

The result is $b = 3$ and $c = -1$.

Step 3 Substitute 3 for *b* and –1 for *c* in one of the original equations with three variables.

$a + 2b - 4c = 8$	Original equation with three variables
$a + 2(3) - 4(-1) = 8$	Replace *b* with 3 and *c* with –1.
$a + 6 + 4 = 8$	Multiply.
$a = -2$	Simplify.

The solution is (–2, 3, –1). You can check this solution in the other two original equations.

Example 2 Infinite Solutions

Solve the system of equations.

$2x + 4y - 2z = 10$
$-3x - 6y + 3z = -15$
$4x - 4y + 8z = 20$

Eliminate x in the first two equations.

$2x + 4y - 2z = 10$	Multiply by 3.	$6x + 12y - 6z = 30$	
$-3x - 6y + 3z = -15$	Multiply by 2.	(+) $-6x - 12y + 6z = -30$	Add the equations.
		$0 = 0$	

The equation $0 = 0$ is always true. This indicates that the first two equations represent the same plane. Check to see if this plane intersects the third plane.

$2x + 4y - 2z = 10$	Multiply by 2.	$4x + 8y - 4z = 20$	
$4x - 4y + 8z = 20$		(–) $4x - 4y + 8z = 20$	Subtract the equations.
		$12y - 12z = 0$	
		$y - z = 0$	Divide each side by the GCF, 12

A solution does exist, and the planes intersect in the line $y - z = 0$. So, there are an infinite number of solutions.

Example 3 No Solution

Solve the system of equations.

$x + y - z = 3$
$-2x - 2y + 2z = -14$
$3x - 4y + 5z = 8$

Eliminate x in the first two equations.

$x + y - z = 3$	Multiply by 2.	$2x + 2y - 2z = 6$	
$-2x - 2y + 2z = -14$		(+) $-2x - 2y + 2z = -14$	Add the equations.
		$0 = -8$	

The equation $0 = -8$ is never true. So there is no solution of this system.

Example 4 Write and Solve a System of Equations

ENTERTAINMENT Scott is in charge of purchasing a variety of tickets to a baseball game to be given to employees who met or exceeded their sales quotas for the past month. There are 3 types of tickets available for the game at a cost of $25 a piece, $35 a piece, or $50 a piece. He knows that he needs to buy 40 tickets in all and can spend $1670. He wants to buy twice as many $35 tickets as $25 tickets. How many of each type of ticket should he buy?

Explore Read the problem and define the variables.
x = the number of $25 tickets
y = the number of $35 tickets
z = the number of $50 tickets

Plan Scott has $1670 to spend on the tickets.

$25x + 35y + 50z = 1670$

The number of tickets should be 40.
$x + y + z = 40$

He wants to buy twice as many \$35 tickets as \$25 tickets.
$2x = y$

Solve Substitute $2x$ for y in each of the first two equations.

$25x + 35y + 50z = 1670$	First equation
$25x + 35(2x) + 50z = 1670$	Replace y with $2x$.
$25x + 70x + 50z = 1670$	Multiply.
$95x + 50z = 1670$	Simplify.

$x + y + z = 40$	Second equation
$x + (2x) + z = 40$	Replace y with $2x$.
$3x + z = 40$	Simplify.

Now solve the system of two equations in two variables.

$95x + 50z = 1670$		$95x + 50z = 1670$
$3x + z = 40$	Multiply by 50.	$(-)\ \underline{150x + 50z = 2000}$
		$-55x = -330$
		$x = 6$

Substitute 6 for x in the equation $2x = y$.

$2(6) = y$	$x = 6$
$12 = y$	Multiply.

Substitute 6 for x and 12 for y in the equation $x + y + z = 40$.

$6 + 12 + z = 40$	$x = 6, y = 12$
$18 + z = 40$	Add.
$z = 22$	Subtract 18 from each side.

So, Scott should buy 6 \$25–tickets, 12 \$35–tickets, and 22 \$50 tickets.

Examine Check to see if all the criteria are met.

The total number of tickets is 40
$6 + 12 + 22 = 40$ ✓

The total amount of money spent is \$1670.

$6(25) + 12(35) + 22(50) \stackrel{?}{=} 1670$
$150 + 420 + 1100 = 1670$ ✓

The number of \$35 tickets is twice the number of \$25 tickets.

$12 \stackrel{?}{=} 2(6)$
$12 = 12$ ✓