

Lesson 4-4

Example 1 Translate a Figure

Find the coordinates of the vertices of the image of quadrilateral $ABCD$ with $A(-4, 1)$, $B(-2, 3)$, $C(0, 0)$, and $D(-4, -3)$, if it is moved 5 units to the right and 3 units down. Then graph $ABCD$ and its image $A'B'C'D'$.

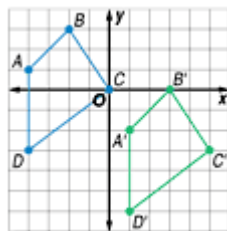
Write the vertex matrix for quadrilateral $ABCD$.

$$\begin{bmatrix} -4 & -2 & 0 & -4 \\ 1 & 3 & 0 & -3 \end{bmatrix}$$

To translate the quadrilateral 5 units to the right, add 5 to each x -coordinate. To translate the figure 3 units down, add -3 to each y -coordinate. This can be done by adding the translation matrix $\begin{bmatrix} 5 & 5 & 5 & 5 \\ -3 & -3 & -3 & -3 \end{bmatrix}$ to the vertex matrix of $ABCD$.

$$\begin{array}{ccc} \text{vertex matrix} & & \text{translation} \\ \text{of } ABCD & & \text{matrix} \\ \begin{bmatrix} -4 & -2 & 0 & -4 \\ 1 & 3 & 0 & -3 \end{bmatrix} & + & \begin{bmatrix} 5 & 5 & 5 & 5 \\ -3 & -3 & -3 & -3 \end{bmatrix} = & \begin{bmatrix} 1 & 3 & 5 & 1 \\ -2 & 0 & -3 & -6 \end{bmatrix} \\ \text{vertex matrix} & & \\ \text{of } A'B'C'D' & & \end{array}$$

The coordinates of $A'B'C'D'$ are $A'(1, -2)$, $B'(3, 0)$, $C'(5, -3)$, $D'(1, -6)$. Graph the preimage and the image. The two quadrilaterals have the same size and shape.



Example 2 Find a Translation Matrix

Short Response Test Item

$\Delta X'Y'Z'$ is the result of a translation of ΔXYZ . A table of the translations is shown. Find the coordinates of Y and Z' .

| ΔXYZ | $\Delta X'Y'Z'$ |
|--------------|-----------------|
| $X(-3, 2)$ | $X'(4, 7)$ |
| Y | $Y'(3, 0)$ |
| $Z(2, -1)$ | Z' |

Read the Test Item

- You are given the coordinates of the preimage and image of point X . Use this information to find the translation matrix. Then you can use the translation matrix to find the coordinates of Y and Z' .

Solve the Test Item

- Write a matrix equation. Let (a, b) represent the coordinates of Y and let (c, d) represent the coordinates of Z' .

$$\begin{bmatrix} -3 & a & 2 \\ 2 & b & -1 \end{bmatrix} + \begin{bmatrix} x & x & x \\ y & y & y \end{bmatrix} = \begin{bmatrix} 4 & 3 & c \\ 7 & 0 & d \end{bmatrix}$$

$$\begin{bmatrix} -3+x & a+x & 2+x \\ 2+y & b+y & -1+y \end{bmatrix} = \begin{bmatrix} 4 & 3 & c \\ 7 & 0 & d \end{bmatrix}$$

- Since these two matrices are equal, corresponding elements are equal.

Solve an equation for x .

$$\begin{aligned} -3 + x &= 4 \\ x &= 7 \end{aligned}$$

Solve an equation for y .

$$\begin{aligned} 2 + y &= 7 \\ y &= 5 \end{aligned}$$

- Use the values for x and y to find the values for $Y(a, b)$ and $Z'(c, d)$.

$$\begin{array}{rclcl} a + x = 3 & b + y = 0 & 2 + x = c & -1 + y = d \\ a + 7 = 3 & b + 5 = 0 & 2 + 7 = c & -1 + 5 = d \\ a = -4 & b = -5 & 9 = c & 4 = d \end{array}$$

So the coordinates of X are $(-4, -5)$ and the coordinates for Z' are $(9, 4)$.

Example 3 Dilation

Quadrilateral $DEFG$ has vertices $D(1, 2)$, $E(4, 1)$, $F(3, -1)$, and $G(0, 0)$. Dilate quadrilateral $DEFG$ so that its perimeter is two and one-half times the original perimeter. What are the coordinates of the vertices of quadrilateral $D'E'F'G'$?

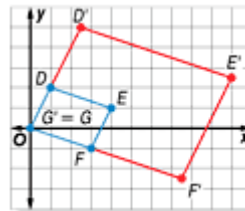
If the perimeter of a figure is two and one-half times the original perimeter, then the lengths of the sides of the figure will be two and one-half times the measure of the original lengths.

Multiply the vertex matrix by the scalar 2.5.

$$2.5 \begin{bmatrix} 1 & 4 & 3 & 0 \\ 2 & 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2.5 & 10 & 7.5 & 0 \\ 5 & 2.5 & -2.5 & 0 \end{bmatrix}$$

The coordinates of the vertices of quadrilateral $D'E'F'G'$ are $D'(2.5, 5)$, $E'(10, 2.5)$, $F'(7.5, -2.5)$, and $G'(0, 0)$.

Graph quadrilateral $DEFG$ and quadrilateral $D'E'F'G'$. The quadrilaterals are not congruent. The image has sides that are two and one-half times the length of the original figure.

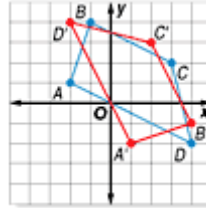


Example 4 Reflection

Find the coordinates of the vertices of the image of quadrilateral $ABCD$ with $A(-2, 1)$, $B(-1, 4)$, $C(3, 2)$, and $D(4, -2)$ after a reflection across the line $y = x$.

Write the ordered pairs as a vertex matrix. Then multiply the vertex matrix by the reflection matrix for the line $y = x$.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & -1 & 3 & 4 \\ 1 & 4 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 2 & -2 \\ -2 & -1 & 3 & 4 \end{bmatrix}$$



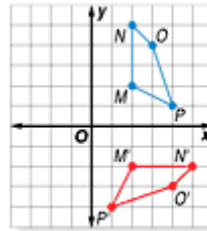
The coordinates of the vertices of $A'B'C'D'$ are $A'(1, -2)$, $B'(4, -1)$, $C'(2, 3)$, and $D'(-2, 4)$. Notice that the preimage and image are congruent. Both figures have the same size and shape.

Example 5 Rotation

Find the coordinates of the vertices of the image of quadrilateral $MNOP$ with $M(2, 2)$, $N(2, 5)$, $O(3, 4)$, and $P(4, 1)$ after it is rotated 270° counterclockwise about the origin.

Write the ordered pairs in a vertex matrix. Then multiply the vertex matrix by the rotation matrix.

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 3 & 4 \\ 2 & 5 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 4 & 1 \\ -2 & -2 & -3 & -4 \end{bmatrix}$$



The coordinates of the vertices of quadrilateral $M'N'O'P'$ are $M'(2, -2)$, $N'(5, -2)$, $O'(4, -3)$, and $P'(1, -4)$. The image is congruent to the preimage.