

Lesson 4–5

Example 1 Second-Order Determinant

Find the value of each determinant.

a. $\begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix}$

$$\begin{aligned} \begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} &= 1(-3) - (-2)(2) && \text{Definition of determinant} \\ &= -3 - (-4) && \text{Multiply.} \\ &= 1 && \text{Simplify.} \end{aligned}$$

b. $\begin{vmatrix} 9 & 6 \\ -6 & -4 \end{vmatrix}$

$$\begin{aligned} \begin{vmatrix} 9 & 6 \\ -6 & -4 \end{vmatrix} &= 9(-4) - 6(-6) && \text{Definition of determinant} \\ &= -36 - (-36) && \text{Multiply.} \\ &= 0 && \text{Simplify.} \end{aligned}$$

Example 2 Expansion by Minors

Evaluate $\begin{vmatrix} 0 & -2 & 3 \\ 1 & -2 & -1 \\ -3 & 3 & 4 \end{vmatrix}$ using expansion by minors.

Decide which row of elements to use for the expansion. For this example, use the second row.

$$\begin{aligned} \begin{vmatrix} 0 & -2 & 3 \\ 1 & -2 & -1 \\ -3 & 3 & 4 \end{vmatrix} &= -1 \begin{vmatrix} -2 & 3 \\ 3 & 4 \end{vmatrix} + (-2) \begin{vmatrix} 0 & 3 \\ -3 & 4 \end{vmatrix} - (-1) \begin{vmatrix} 0 & -2 \\ -3 & 3 \end{vmatrix} && \text{Expansion by minors.} \\ &= -1[-8 - 9] - 2[0 - (-9)] + 1[0 - 6] && \text{Evaluate } 2 \times 2 \text{ determinants.} \\ &= -1(-17) - 2(9) + 1(-6) \\ &= 17 - 18 - 6 && \text{Multiply.} \\ &= -7 && \text{Simplify.} \end{aligned}$$

Example 3 Use Diagonals

Evaluate $\begin{vmatrix} 4 & -3 & -1 \\ 1 & 0 & -2 \\ -5 & 2 & 0 \end{vmatrix}$ using diagonals.

Step 1 Rewrite the first two columns to the right of the determinant.

$$\begin{vmatrix} 4 & -3 & -1 & 4 & -3 \\ 1 & 0 & -2 & 1 & 0 \\ -5 & 2 & 0 & -5 & 2 \end{vmatrix}$$

Step 2 Find the products of the elements of the diagonals.

$$\begin{vmatrix} 4 & -3 & -1 & 4 & -3 \\ 1 & 0 & -2 & 1 & 0 \\ -5 & 2 & 0 & -5 & 2 \end{vmatrix} \quad \begin{matrix} 0 & -16 & 0 \\ 4 & -3 & -1 & 4 & -3 \\ 1 & 0 & -2 & 1 & 0 \\ -5 & 2 & 0 & -5 & 2 \end{matrix}$$

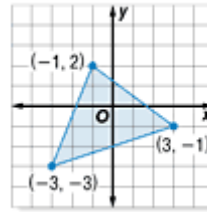
$$\begin{matrix} 0 & -30 & -2 \end{matrix}$$

Step 3 Add the bottom products and subtract the top products.

$$0 + (-30) + (-2) - (0) - (-16) - (0) = -16$$

The value of the determinant is -16 .

Example 4 Area of a Triangle
GEOMETRY Find the area of a triangle whose vertices are located at $(-3, -3)$, $(-1, 2)$, and $(3, -1)$.



Assign values to $a, b, c, d, e,$ and f and substitute them into the Area Formula. Then evaluate.

$$A = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

Area Formula

$$= \frac{1}{2} \begin{vmatrix} -3 & -3 & 1 \\ -1 & 2 & 1 \\ 3 & -1 & 1 \end{vmatrix}$$

$$(a, b) = (-3, -3), (c, d) = (-1, 2), (e, f) = (3, -1)$$

$$= \frac{1}{2} \left[-3 \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - (-3) \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} \right]$$

Expansion by minors

$$= \frac{1}{2} [-3(2 + 1) + 3(-1 - 3) + 1(1 - 6)]$$

Evaluate 2×2 determinants.

$$= \frac{1}{2} [-9 - 12 - 5]$$

Multiply.

$$= \frac{1}{2} [-26] \text{ or } -13$$

Simplify.

Remember that the area of a triangle is the absolute value of A . Thus, the area is $|-13|$ or 13 square units.