Lesson 5–8

Example 1 Solve a Radical Equation

Solve $-10 + \sqrt{c-1} = 2$

$-10 + \sqrt{c-1} = 2$	Original equation
$\sqrt{c-1} = 12$	Add 10 to each side to isolate the radical.
$(\sqrt{c-1})^2 = (12)^2$	Square each side to eliminate the radical.
$c - 1 = 144$	Find the squares.
$c = 145$	Add 1 to each side.

Check

$-10 + \sqrt{c-1} = 2$	Original equation
$-10 + \sqrt{145-1} \stackrel{?}{=} 2$	Replace *c* with 145.
$2 = 2$ ✓	Simplify.

The solution checks. The solution is 2.

Example 2 Extraneous Solution

Solve $5 + \sqrt{x-4} = 2$.

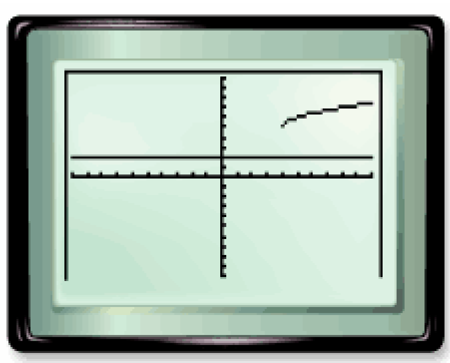
$5 + \sqrt{x-4} = 2$	Original equation
$\sqrt{x-4} = -3$	Subtract 3 from each side to isolate the radical.
$(\sqrt{x-4})^2 = (-3)^2$	Square each side.
$x - 4 = 9$	Find the squares.
$x = 13$	Add 4 to each side.

Check

$$5 + \sqrt{x-4} = 2$$
$$5 + \sqrt{13-4} \stackrel{?}{=} 2$$
$$5 + 3 \stackrel{?}{=} 2$$
$$8 \neq 2$$

The solution does not check, so the equation has no real solution.

The graphing calculator screen shows the graphs of $y = 5 + \sqrt{x-4}$ and $y = 2$. The graphs do not intersect, which confirms that there is no solution.



Example 3 Cube Root Equation

Solve $6\sqrt[3]{2x-1} - 25 = 5$.

In order to remove the cube root, you must first isolate it and then raise each side of the equation to the third power.

$6\sqrt[3]{2x-1} - 25 = 5$	Original equation
$6\sqrt[3]{2x-1} = 30$	Add 25 to each side.
$\sqrt[3]{2x-1} = 5$	Divide each side by 6.
$(\sqrt[3]{2x-1})^3 = (5)^3$	Cube each side.
$2x - 1 = 125$	Evaluate the cubes.
$2x = 126$	Add 1 to each side.
$x = 63$	Divide each side by 2.

Check

$6\sqrt[3]{2x-1} - 25 = 5$	Original equation
$6\sqrt[3]{2(63)-1} - 25 \stackrel{?}{=} 5$	Replace x with 63.
$6\sqrt[3]{125} - 25 \stackrel{?}{=} 5$	Simplify.
$6(5) - 25 \stackrel{?}{=} 5$	The cube root of 125 is 5.
$5 = 5$ ✓	Subtract.

The solution is 63.

Example 4 Radical Inequality

Solve $\sqrt{5x+4}-6 \le 2$.

Since the radicand of a square root must be greater than or equal to zero, first solve $5x + 4 \ge 0$ to identify the values of x for which the left side of the given inequality is defined.

$$5x + 4 \ge 0$$
$$5x \ge -4$$
$$x \ge -\frac{4}{5}$$

Now solve $\sqrt{5x+4}-6 \le 2$.

$\sqrt{5x+4}-6 \le 2$	Original inequality
$\sqrt{5x+4} \le 8$	Isolate the radical.
$5x + 4 \le 64$	Eliminate the radical.
$5x \le 60$	Subtract 4 from each side.
$x \le 12$	Divide each side by 5.

It appears that $-\frac{4}{5} \le x \le 12$. You can test some x values to confirm the solution. Let $f(x) = \sqrt{5x+4}-6$. Use three test value: one less than $-\frac{4}{5}$, one between $-\frac{4}{5}$ and 12, and one greater than 12. Organize the test values in a table.

$x = -1$	$x = 0$	$x = 14$
$f(-1) = \sqrt{5(-1)+4}-6$ $= \sqrt{-1}-6$ Since $\sqrt{-1}$ is not a real number, the inequality is not satisfied.	$f(0) = \sqrt{5(0)+4}-6$ $= -4$ Since $-4 \le 2$, the inequality is satisfied.	$f(14) = \sqrt{5(14)+4}-6$ ≈ 2.60 Since $2.60 \not\le 2$, the inequality is not satisfied.

The solution checks. Only values in the interval $-\frac{4}{5} \le x \le 12$ satisfy the inequality. You can summarize the solution with a number line.