Lesson 6–4

Example 1 Equation with Rational Roots

Solve $2x^2 - 36x + 162 = 32$ by using the Square Root Property.

$2x^2 - 36x + 162 = 32$	Original equation
$2(x^2 - 18x + 81) = 2(16)$	Factor out the GCF.
$x^2 - 18x + 81 = 16$	Divide each side by 2.
$(x - 9)^2 = 16$	Factor the perfect trinomial square.
$x - 9 = \pm\sqrt{16}$	Square Root Property
$x - 9 = \pm 4$	$\sqrt{16} = 4$
$x = 9 \pm 4$	Add 9 to each side.
$x = 9 + 4$ or $x = 9 - 4$	Write as two equations.
$x = 13$ $\quad$ $x = 5$	Solve each equation.

The solution set is {5, 13}. You can check this result by using factoring to solve the original equation.

Example 2 Equation with Irrational Roots

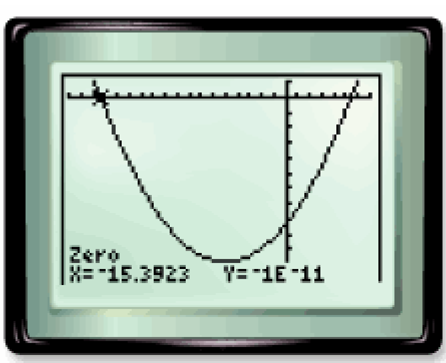
Solve $x^2 + 10x + 25 = 108$ by using the Square Root Property.

$x^2 + 10x + 25 = 108$	Original equation
$(x + 5)^2 = 108$	Factor the perfect square trinomial.
$x + 5 = \pm\sqrt{108}$	Square Root Property
$x = -5 \pm 6\sqrt{3}$	Add –5 to each side; $\sqrt{108} = 6\sqrt{3}$
$x = -5 + 6\sqrt{3}$ or $x = -5 - 6\sqrt{3}$	Write as two equations.
$x \approx 5.4$ $\quad$ $x \approx -15.4$	Use a calculator.

The exact solutions of this equation are $-5 - 6\sqrt{3}$ and $-5 + 6\sqrt{3}$. The approximate solutions are –15.4 and 5.4. Check these results by finding and graphing the related quadratic function.

$x^2 + 10x + 25 = 108$	Original equation
$x^2 + 10x - 83 = 0$	Subtract 108 from each side.
$y = x^2 + 10x - 83$	Related quadratic function.

CHECK: Use the ZERO function of a graphing calculator. The approximate zeros of the related function are –15.4 and 5.4.



Example 3 Complete the Square

Find the value of *c* that makes $x^2 - \frac{5}{3}x + c$ a perfect square. Then write the trinomial as a perfect square.

Step 1 Find one half of $\frac{5}{3}$. $\quad \frac{5}{3} \bullet \frac{1}{2} = \frac{5}{6}$

Step 2 Square the result of Step 1. $\quad \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

Step 3 Add the result of Step 2 to $x^2 - \frac{5}{3}x$ $\quad x^2 - \frac{5}{3}x + \frac{25}{36}$

The trinomial $x^2 - \frac{5}{3}x + \frac{25}{36}$ can be written as $\left(x - \frac{5}{6}\right)^2$.

Example 4 Solve an Equation by Completing the Square

Solve $x^2 - 7x - 44 = 0$ by completing the square.

$x^2 - 7x - 44 = 0$ — Notice that $x^2 - 7x - 44$ is not a perfect square.

$x^2 - 7x = 44$ — Rewrite so the left side is of the form $x^2 + bx$.

$x^2 - 7x + \frac{49}{4} = 44 + \frac{49}{4}$ — Since $\left(\frac{7}{2}\right)^2 = \frac{49}{4}$, add $\frac{49}{4}$ to each side.

$\left(x - \frac{7}{2}\right)^2 = \frac{225}{4}$ — Write the left side as a perfect square by factoring.

$x - \frac{7}{2} = \pm\sqrt{\frac{225}{4}}$ — Square Root Property

$x - \frac{7}{2} = \pm\frac{15}{2}$ — $\sqrt{\frac{225}{4}} = \frac{15}{2}$

$x = \frac{7}{2} \pm \frac{15}{2}$ — Add $\frac{7}{2}$ to each side.

$x = \frac{7}{2} + \frac{15}{2}$ or $x = \frac{7}{2} - \frac{15}{2}$ — Write as two equations.

$x = 11$ $\quad x = -4$ — Solve each equation.

The solution set is {–4, 11}. You can check this result by using factoring to solve the original equation.

Example 5 **Equation with $a \neq 1$**

Solve $3x^2 + 4x - 7 = 0$ by completing the square.

$3x^2 + 4x - 7 = 0$	Notice that $3x^2 + 4x - 7$ is not a perfect square.
$x^2 + \frac{4}{3}x - \frac{7}{3} = 0$	Divide by the coefficient of the quadratic term, 3.
$x^2 + \frac{4}{3}x = \frac{7}{3}$	Add $\frac{7}{3}$ to each side.
$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{7}{3} + \frac{4}{9}$	Since $\left(\left(\frac{4}{3}\right) \div 2\right)^2 = \frac{4}{9}$, add $\frac{4}{9}$ to each side.
$\left(x + \frac{2}{3}\right)^2 = \frac{25}{9}$	Write the left side as a perfect square by factoring.
	Simplify the right side.
$x + \frac{2}{3} = \pm\frac{5}{3}$	Square Root Property
$x = -\frac{2}{3} \pm \frac{5}{3}$	Add $-\frac{2}{3}$ to each side.
$x = -\frac{2}{3} + \frac{5}{3}$ or $x = -\frac{2}{3} - \frac{5}{3}$	Write as two equations.
$x = 1$ $\quad$ $x = -\frac{7}{3}$	Simplify.

The solution set is $\left\{-\frac{7}{3}, 1\right\}$.

Example 6 **Equation with Complex Solutions**

Solve $4x^2 - 2x + 7 = 0$ by completing the square.

$4x^2 - 2x + 7 = 0$	Notice that $4x^2 - 2x + 7$ is not a perfect square.
$x^2 - \frac{1}{2}x + \frac{7}{4} = 0$	Divide by the coefficient of the quadratic term, 4.
$x^2 - \frac{1}{2}x = -\frac{7}{4}$	Rewrite so the left side is of the form $x^2 + bx$.
$x^2 - \frac{1}{2}x + \frac{1}{16} = -\frac{7}{4} + \frac{1}{16}$	Since $\left(-\frac{1}{2} \div 2\right)^2 = \frac{1}{16}$, add $\frac{1}{16}$ to each side.
$\left(x - \frac{1}{4}\right)^2 = -\frac{27}{16}$	Write the left side as a perfect square by factoring.
	Simplify the right side.
$x - \frac{1}{4} = \pm\sqrt{-\frac{27}{16}}$	Square Root Property
$x - \frac{1}{4} = \pm\frac{3i\sqrt{3}}{4}$	$\sqrt{-1} = i$
$x = \frac{1}{4} \pm \frac{3i\sqrt{3}}{4}$	Add $\frac{1}{4}$ to each side.

The solution set is $\left\{\frac{1}{4}+\frac{3\boldsymbol{i}\sqrt{3}}{4}, \frac{1}{4}-\frac{3\boldsymbol{i}\sqrt{3}}{4}\right\}$. Notice that these are imaginary solutions.

CHECK: A graph of the related function shows that the equation has no real solutions since the graph has no *x*–intercepts. Imaginary solutions must be checked algebraically by substituting them back in the original equation.