Lesson 6–5

Example 1 Two Rational Roots

Solve $2x^2 - x = 15$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$2x^2 - x = 15 \quad \rightarrow \quad \begin{matrix} ax^2 + bx + c = 0 \\ \downarrow \quad \downarrow \quad \downarrow \\ 2x^2 - 1x - 15 = 0 \end{matrix}$$

Then, substitute these values into the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-15)}}{2(2)}$ Replace a with 2, b with –1, and c with –15.

$x = \frac{1 \pm \sqrt{1 + 120}}{4}$ Simplify.

$x = \frac{1 \pm \sqrt{121}}{4}$ Simplify.

$x = \frac{1 \pm 11}{4}$ $\sqrt{121} = 11$

$x = \frac{1 + 11}{4}$ or $x = \frac{1 - 11}{4}$ Write as two equations.

$= 3$ $\quad = -2.5$ Simplify.

The solutions are –2.5 and 3. Check by substituting each of these values into the original equation.

Example 2 One Rational Root

Solve $2x^2 - 52x + 338 = 0$ by using the Quadratic Formula.

Identify a, b, and c. Then substitute these values into the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

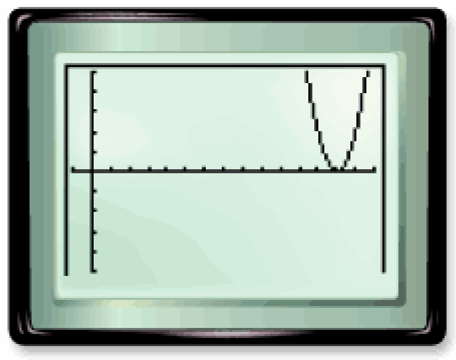
$x = \frac{-(-52) \pm \sqrt{(-52)^2 - 4(2)(338)}}{2(2)}$ Replace a with 2, b with –52, and c with 338.

$x = \frac{52 \pm \sqrt{0}}{4}$ Simplify.

$x = \frac{52}{4}$ or 13 $\sqrt{0} = 0$

The solution is 13.

CHECK: A graph of the related function shows that there is one solution at $x = 13$.



Example 3 Irrational Roots

Solve $5x^2 + x - 1 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(5)(-1)}}{2(5)}$$ Replace *a* with 5, *b* with 1, and *c* with –1.

$$x = \frac{-1 \pm \sqrt{21}}{10}$$ Simplify.

$$x = \frac{-1 + \sqrt{21}}{10} \text{ or } \frac{-1 - \sqrt{21}}{10}$$ Rewrite as two equations.

The exact solutions are $\frac{-1 - \sqrt{21}}{10}$ and $\frac{-1 + \sqrt{21}}{10}$. The approximate solutions are –0.6 and 0.4.

CHECK: Check these results by graphing the related quadratic function, $y = 5x^2 + x - 1$. Using the ZERO function of a graphing calculator, the approximate zeros of the related function are –0.6 and 0.4.

Example 4 Complex Roots

Solve $7x^2 + 3 = 2x$ by using the Quadratic Formula.

Rewrite as $7x^2 - 2x + 3 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(3)}}{2(7)}$$ Replace a with 7, b with –2, and c with 3.

$$x = \frac{2 \pm \sqrt{-80}}{2(7)}$$ Simplify.

$$x = \frac{2 \pm 4\boldsymbol{i}\sqrt{5}}{14}$$ $\sqrt{-80} = \sqrt{16(5)(-1)}$ or $4\boldsymbol{i}\sqrt{5}$

$$x = \frac{1 \pm 2\boldsymbol{i}\sqrt{5}}{7}$$ Simplify.

The solutions are the complex numbers $\frac{1 + 2\boldsymbol{i}\sqrt{5}}{7}$ and $\frac{1 - 2\boldsymbol{i}\sqrt{5}}{7}$.

A graph of the related function shows that the solutions are complex, but it cannot help you find them.

CHECK: To check complex solutions, you must substitute them into the original equation. The check for $\frac{1 + 2\boldsymbol{i}\sqrt{5}}{7}$ is shown below.

$$7x^2 + 3 = 2x$$ Original equation

$$7\left(\frac{1+2\boldsymbol{i}\sqrt{5}}{7}\right)^2 + 3 \stackrel{?}{=} 2\left(\frac{1+2\boldsymbol{i}\sqrt{5}}{7}\right)$$ $x = \frac{1 + 2\boldsymbol{i}\sqrt{5}}{7}$

$$7\left(\frac{1+4\boldsymbol{i}\sqrt{5}+4\boldsymbol{i}^2(5)}{49}\right) + 3 \stackrel{?}{=} 2\left(\frac{1+2\boldsymbol{i}\sqrt{5}}{7}\right)$$ Sum of a square

$$\frac{-19 + 4\boldsymbol{i}\sqrt{5}}{7} + 3 \stackrel{?}{=} \frac{2 + 4\boldsymbol{i}\sqrt{5}}{7}$$ Simplify.

$$\frac{2 + 4\boldsymbol{i}\sqrt{5}}{7} = \frac{2 + 4\boldsymbol{i}\sqrt{5}}{7} \checkmark$$ Simplify.

Example 5 Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $15x + 10x^2 - 25 = 0$

$a = 10$, $b = 15$, $c = -25$

$b^2 - 4ac = 15^2 - 4(10)(-25)$

$= 225 + 1000$

$= 1225$ or 35^2

The discriminant is 1225, which is a perfect square. Therefore, there are two rational roots.

b. $7x^2 + 14x + 7 = 0$

$a = 7$, $b = 14$, $c = 7$

$b^2 - 4ac = 14^2 - 4(7)(7)$

$= 196 - 196$

$= 0$

The discriminant is 0, so there is one rational root.

c. $6x^2 + 2x + 5 = 0$

$a = 6$, $b = 2$, $c = 5$

$b^2 - 4ac = 2^2 - 4(6)(5)$

$= 4 - 120$

$= -116$

The discriminant is negative, so there are two complex roots.

d. $11x^2 - 10x - 2 = 0$

$a = 11$, $b = -10$, $c = -2$

$b^2 - 4ac = (-10)^2 - 4(11)(-2)$

$= 100 + 88$

$= 188$

The discriminant is 188, which is not a perfect square. Therefore, there are two irrational roots.