Lesson 7–1

Example 1 **Find Degree and Leading Coefficients**

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $-2a^3 + a - \frac{1}{a^2}$

This is not a polynomial. The term $\frac{1}{a^2}$ cannot be written in the form x^n, where n is a nonnegative integer.

b. $b^5 + b^2 - 8$

This is a polynomial in one variable. The degree is 5, and the leading coefficient is 1.

c. $mn + 3m - 7n$

This is not a polynomial in one variable. It contains two variables, m and n.

d. $11 - 5c + 2c^6 - 4c^3$

Rewrite the expression so the powers of c are in decreasing order.

$$2c^6 - 4c^3 - 5c + 11$$

This is a polynomial in one variable. The degree is 6, and the leading coefficient is 2.

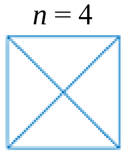
Example 2 **Evaluate a Polynomial Function**

GEOMETRY In a polygon with n sides, for $n > 3$, the number of diagonals can be found by using the function $d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$, where $d(n)$ is the number of diagonals and n is the number of sides.

a. Draw polygons for n = 4, 5, 6, and 7. Show that the polynomial function $d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$ gives the total number of diagonals for each figure.

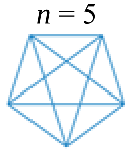
First, draw the polygons and count the number of diagonals.

$n = 4$



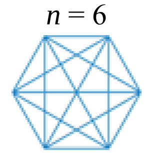
diagonals = 2

$n = 5$



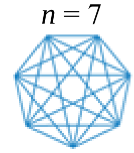
diagonals = 5

$n = 6$



diagonals = 9

$n = 7$



diagonals = 14

Next, find the values of $d(4)$, $d(5)$, $d(6)$, and $d(7)$.

$$d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$$
$$d(4) = \frac{1}{2}(4)^2 - \frac{3}{2}(4)$$
$$= 8 - 6$$
$$= 2$$

$$d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$$
$$d(5) = \frac{1}{2}(5)^2 - \frac{3}{2}(5)$$
$$= 12.5 - 7.5$$
$$= 5$$

$$d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$$
$$d(6) = \frac{1}{2}(6)^2 - \frac{3}{2}(6)$$
$$= 18 - 9$$
$$= 9$$

$$d(n) = \frac{1}{2}n^2 - \frac{3}{2}n$$
$$d(7) = \frac{1}{2}(7)^2 - \frac{3}{2}(7)$$
$$= 24.5 - 10.5$$
$$= 14$$

You can see that the number of diagonals for each figure in the diagrams is the same as the number of diagonals $d(n)$ found by evaluating the function.

b. Find the number of diagonals for a decagon (a ten–sided polygon).

$nd(n) = \frac{1}{2}n^2 - \frac{3}{2}n$ Given function

$d(10) = \frac{1}{2}(10)^2 - \frac{3}{2}(10)$ Replace n with 10.

$= 50 - 15$ Evaluate.

$= 35$ Simplify.

Example 3 Functional Values of Variables

a. Find $p(-2)$ if $p(x) = -x^4 - x^3 + 6$.

$p(x) = -x^4 - x^3 + 6$ Original function

$p(-2) = -(-2)^4 - (-2)^3 + 6$ Replace x with -2.

$= -16 - (-8) + 6$ Simplify.

$= -2$ Simplify.

b. Find $-3[q(4b - 1)]$ if $q(x) = x^2 - x + 1$.

First, evaluate $q(4b - 1)$ by replacing x in $q(x)$ with $4b - 1$.

$q(x) = x^2 - x + 1$ Original function

$q(4b - 1) = (4b - 1)^2 - (4b - 1) + 1$ Replace x with $4b - 1$.

$= 16b^2 - 8b + 1 - 4b + 1 + 1$ Evaluate $(4b - 1)^2$ and $-1(4b - 1)$.

$= 16b^2 - 12b + 3$ Simplify.

To evaluate $-3[q(4b - 1)]$, multiply $q(4b - 1)$ by -3.

$-3[q(4b - 1)] = -3(16b^2 - 12b + 3)$ Substitute $16b^2 - 12b + 3$ for $q(4b - 1)$.

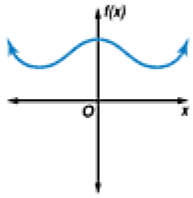
$= -48b^2 + 36b - 9$ Distributive Property

Example 4 Graphs of Polynomial Functions

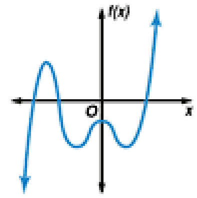
For each graph:

- **Describe the end behavior.**
- **Determine whether it represents an odd–degree or an even–degree polynomial function.**
- **State the number of real zeros.**

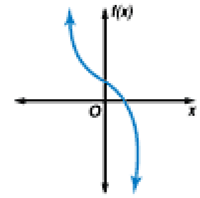
a.



b.



c.



a.
- As $x \to +\infty$, $f(x) \to +\infty$. As $x \to -\infty$, $f(x) \to +\infty$.
- It is an even–degree polynomial function.
- The graph does not intersect the *x*–axis, so it has no real zeros.

b.
- As $x \to +\infty$, $f(x) \to +\infty$. As $x \to -\infty$, $f(x) \to -\infty$.
- It is an odd–degree polynomial function.
- The graph intersects the *x*–axis at three points, so the function has three real zeros.

c.
- As $x \to -\infty$, $f(x) \to +\infty$. As $x \to +\infty$, $f(x) \to -\infty$.
- It is an odd–degree polynomial function.
- The graph intersects the *x*–axis at one point, so the function has one real zero.