

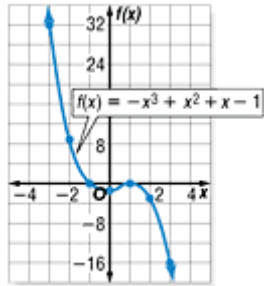
Lesson 7-2

Example 1 Graph a Polynomial Function

Graph the polynomial function $f(x) = -x^3 + x^2 + x - 1$ by making a table of values.

Make a table for several values of x and plot the points. Connect the points with a smooth curve.

x	$f(x)$
-3	32
-2	9
-1	0
0	-1
1	0
2	-3
3	-16



This is an odd-degree polynomial with a negative leading coefficient, so $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$, and $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$. Notice that the graph intersects the x -axis at two points, indicating there are two real zeros for this function.

Example 2 Locate Zeros of a Function

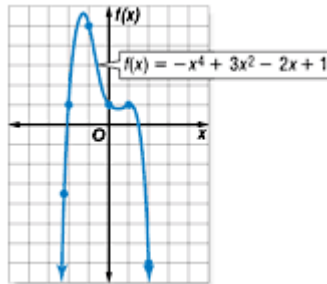
Determine the values of x between which the real zeros of the function $f(x) = -x^4 + 3x^2 - 2x + 1$ are located. Then draw the graph.

Make a table of values. Since $f(x)$ is a fourth-degree polynomial function, it could have up to 4 real zeros. Look at the values of $f(x)$ to locate the zeros. Then use the points to sketch a graph of the function.

x	$f(x)$
-3	-47
-2	1
-1	5
0	1
1	1
2	-7
3	-59

change in signs

change in signs



The changes in sign indicate that there are zeros between $x = -3$ and $x = -2$, and between $x = 1$ and $x = 2$.

Example 3 Maximum and Minimum Points

Graph $f(x) = -x^5 + x^3 - x^2 + 2$. Estimate the x -coordinates at which the relative maxima and relative minima occur.

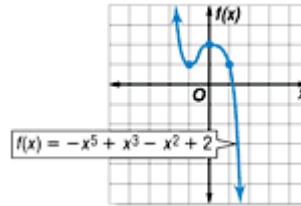
Make a table of values and graph the equation.

x	$f(x)$
-3	209
-2	22
-1	1
0	2
1	1
2	-26
3	-223

← indicates a relative minimum

← indicates a relative maximum

zero between 1 and 2



Look at the table of values and the graph.

- The values of $f(x)$ change signs between $x = 1$ and $x = 2$, indicating a zero of the function.
- The value of $f(x)$ at $x = -1$ is less than the surrounding points, so it is a relative minimum.
- The value of $f(x)$ at $x = 0$ is greater than the surrounding points, so it is a relative maximum.

Example 4 Graph a Polynomial Model

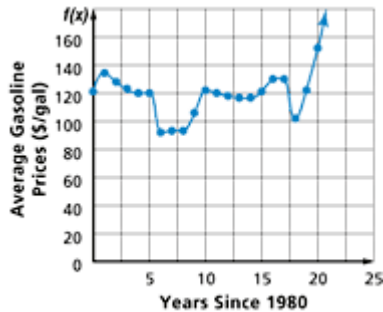
ECONOMICS The table shows the average retail prices of gasoline from 1980 through 2000. The x -values are given as years since 1980, so that 1980 = 0, 1981 = 1, and so on, and the values of $f(x)$ are the prices.

x	$f(x)$		x	$f(x)$		x	$f(x)$		x	$f(x)$
0	122		5	120		10	122		15	121
1	135		6	93		11	120		16	129
2	128		7	96		12	119		17	129
3	123		8	96		13	117		18	112
4	120		9	106		14	117		19	122
									20	153

Source: *World Almanac*

a. Graph the function.

Plot the points from the table.



b. Describe the turning points of the graph and its end behavior.

There are relative maxima at 1, 10, 16–17, and 20. There are relative minima at 6, 13–14, and 18. For the end behavior, as x is increasing, $f(x)$ is increasing, but only from 1999 to 2000.

c. What do the maxima and minima represent in this situation?

The maxima are high prices for gasoline in a particular time period and the minima are low prices for gasoline in a particular time period.