Lesson 7–3

Example 1 Write an Expression in Quadratic Form

Write each expression in quadratic form, if possible.

a. $4x^8 - 25$

$4x^8 - 25 = (2x^4)^2 - 25$ $\quad (x^4)^2 = x^8$

b. $25x^6 - 3x^2 + 1$

This cannot be written in quadratic form since $x^6 \neq (x^2)^2$.

c. $16x^{10} - 8x^5 - 7$

$16x^{10} - 8x^5 - 7 = (4x^5)^2 - 8(x^5) - 7$ $\quad (x^5)^2 = x^{10}$

d. $36x - 4x^{\frac{1}{2}} + 1$

$36x - 4x^{\frac{1}{2}} + 1 = (6x^{\frac{1}{2}})^2 - 4(x^{\frac{1}{2}}) + 1$ $\quad (x^{\frac{1}{2}})^2 = x^1$

Example 2 Solve Polynomial Equations

Solve each equation.

a. $2x^4 - 3x^2 + 1 = 0$

$2x^4 - 3x^2 + 1 = 0$ Original equation

$2(x^2)^2 - 3(x^2) + 1 = 0$ Write the expression on the left side in quadratic form.

$(2x^2 - 1)(x^2 - 1) = 0$ Factor the trinomial.

$(2x^2 - 1)(x - 1)(x + 1) = 0$ Factor the difference of squares.

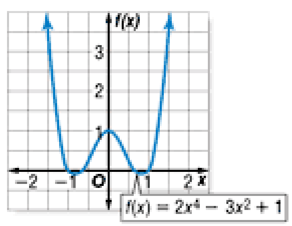
Use the Zero Product Property.

$2x^2 - 1 = 0$ or $x - 1 = 0$ or $x + 1 = 0$

$2x^2 = 1$ $\quad x = 1$ $\quad x = -1$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$



The solutions are $-1, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$, and 1.

CHECK: The graph of $f(x) = 2x^4 - 3x^2 + 1$ shows that the graph intersects the x–axis at $-1, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$, and 1. ✓

b. $x^6 - 64 = 0$

$x^6 - 64 = 0$ Original equation

$(x^3)^2 - 8^2 = 0$ This is the difference of two squares.

$(x^3 - 8)(x^3 + 8) = 0$ Factor the difference of two squares.

$x^3 - 8 = 0$ or $x^3 + 8 = 0$ Zero Product Property

$x^3 = 8$ $\quad x^3 = -8$

$x = 2$ $\quad x = -2$ Take the cube root to solve each equation.

The solutions are –2 and 2. Check this solution by examining the graph of the related function.

Example 3 Solve Equations with Rational Exponents

Solve $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 4 = 0$.

$x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 4 = 0$	Original equation
$(x^{\frac{1}{4}})^2 - 5(x^{\frac{1}{4}}) + 4 = 0$	Write the expression on the left in quadratic form.
$(x^{\frac{1}{4}} - 1)(x^{\frac{1}{4}} - 4) = 0$	Factor the trinomial.
$x^{\frac{1}{4}} - 1 = 0$ or $x^{\frac{1}{4}} - 4 = 0$	Zero Product Property
$x^{\frac{1}{4}} = 1$ $x^{\frac{1}{4}} = 4$	Isolate *x* on one side of the equation.
$(x^{\frac{1}{4}})^4 = 1^4$ $(x^{\frac{1}{4}})^4 = 4^4$	Raise each side to the fourth power.
$x = 1$ $x = 256$	Simplify.

CHECK Substitute each value into the original equation.

$$x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 4 = 0 \qquad x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 4 = 0$$

$$1^{\frac{1}{2}} - 5\left(1^{\frac{1}{4}}\right) + 4 \stackrel{?}{=} 0 \qquad 256^{\frac{1}{2}} - 5\left(256^{\frac{1}{4}}\right) + 4 \stackrel{?}{=} 0$$

$$1 - 5 + 4 \stackrel{?}{=} 0 \qquad 16 - 20 + 4 \stackrel{?}{=} 0$$

$$0 = 0 \ \checkmark \qquad 0 = 0 \ \checkmark$$

The solutions are 1 and 256.

Example 4 Solve Radical Equations

Solve $x = -2\sqrt{x} + 15$.

$x = -2\sqrt{x} + 15$	Original equation
$x + 2\sqrt{x} - 15 = 0$	Rewrite so that one side is zero.
$(\sqrt{x})^2 + 2(\sqrt{x}) - 15 = 0$	Write the expression on the left in quadratic form.

You can use the Quadratic Formula to solve this equation.

$\sqrt{x} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$\sqrt{x} = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-15)}}{2(1)}$	Replace *a* with 1, *b* with 2, and *c* with –15.
$\sqrt{x} = \dfrac{-2 \pm 8}{2}$	Simplify.
$\sqrt{x} = \dfrac{-2 + 8}{2}$ or $\sqrt{x} = \dfrac{-2 - 8}{2}$	Write as two equations.
$\sqrt{x} = 3$ $\sqrt{x} = -5$	Simplify.
$x = 9$	

There is no real number *x* such that $\sqrt{x} = -5$. The only real solution is 9.