Lesson 7–4

Example 1 Synthetic Substitution

If $f(x) = 5x^3 + 2x^2 - x + 4$, find $f(-1)$.

Method 1 Synthetic Substitution

By the Remainder Theorem, $f(-1)$ should be the remainder when you divide the polynomial by $x + 1$.

–1	5	2	–1	4
		–5	3	–2
	5	–3	2	2

The remainder is 2. Thus, by using synthetic substitution, $f(-1) = 2$.

Method 2 Direct Substitution

Replace x with –1.

$f(x) = 5x^3 + 2x^2 - x + 4$ Original function

$f(-1) = 5(-1)^3 + 2(-1)^2 - (-1) + 4$ Replace x with -1.

$= -5 + 2 + 1 + 4$ or 2 Simplify.

By using direct substitution, $f(-1) = 2$.

Example 2 Use the Factor Theorem

Show that $2x - 1$ is a factor of $6x^3 + 23x^2 - 5x - 4$. Then find the remaining factors of the polynomial.

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{(6x^3 + 23x^2 - 5x - 4) \div 2}{(2x - 1) \div 2} = \frac{3x^3 + \frac{23}{2}x^2 - \frac{5}{2}x - 2}{x - \frac{1}{2}}$$

The binomial $x - \frac{1}{2}$ is a factor of the polynomial if $\frac{1}{2}$ is a zero of the polynomial function. Use the Factor Theorem.

$\frac{1}{2}$	3	$\frac{23}{2}$	$-\frac{5}{2}$	–2
		$\frac{3}{2}$	$\frac{13}{2}$	2
	3	13	4	0

Since the remainder is 0, $2x - 1$ is a factor of the polynomial. The polynomial $6x^3 + 23x^2 - 5x - 4$ can be factored as $(2x - 1)(3x^2 + 13x + 4)$. The polynomial $3x^2 + 13x + 4$ is the depressed polynomial. Check to see if this polynomial can be factored.

$3x^2 + 13x + 4 = (3x + 1)(x + 4)$ Factor the trinomial.

So, $6x^3 + 23x^2 - 5x - 4 = (2x - 1)(3x + 1)(x + 4)$. The zeros of this function are -4, $-\frac{1}{3}$, and $\frac{1}{2}$.

Example 3 Find All Factors of a Polynomial

NUMBER THEORY The product of three numbers can be written as the function $P(x) = x^3 + 9x^2 - 40x - 300$. It is known that one of the numbers can be expressed as $x + 5$. Find the other two numbers in terms of x.

You know that one of the numbers in terms of x is $x + 5$. So $x + 5$ is a factor of $P(x)$.

–5	1	9	–40	–300
		–5	–20	300
	1	4	–60	0

The quotient is $x^2 + 4x - 60$. Use this to factor $P(x)$.

$P(x) = x^3 + 9x^2 - 40x - 300$ Product function

$= (x + 5)(x^2 + 4x - 60)$ Factor.

$= (x + 5)(x + 10)(x - 6)$ Factor the trinomial $x^2 + 4x - 60$.

So, the missing numbers are $(x + 10)$ and $(x - 6)$.