Lesson 7–7

Example 1 Add and Subtract Functions

Given $f(x) = x^3 - 5x^2 - 7x$ and $g(x) = -x^2 + 3$, find each function.

a. $(f + g)(x)$

$(f + g)(x) = f(x) + g(x)$	Addition of functions
$= (x^3 - 5x^2 - 7x) + (-x^2 + 3)$	$f(x) = x^3 - 5x^2 - 7x$ and $g(x) = -x^2 + 3$
$= x^3 - 6x^2 - 7x + 3$	Simplify.

b. $(f - g)(x)$

$(f - g)(x) = f(x) - g(x)$	Subtraction of functions
$= (x^3 - 5x^2 - 7x) - (-x^2 + 3)$	$f(x) = x^3 - 5x^2 - 7x$ and $g(x) = -x^2 + 3$
$= x^3 - 4x^2 - 7x - 3$	Simplify.

Example 2 Multiply and Divide Functions

Given $f(x) = 2x^2 + 5x - 12$ and $g(x) = 3x + 12$, find each function.

a. $(f \bullet g)(x)$

$(f \bullet g)(x) = f(x) \bullet g(x)$	Product of functions
$= (2x^2 + 5x - 12)(3x + 12)$	$f(x) = 2x^2 + 5x - 12$ and $g(x) = 3x + 12$
$= 2x^2(3x + 12) + 5x(3x + 12) - 12(3x + 12)$	Distributive Property
$= 6x^3 + 24x^2 + 15x^2 + 60x - 36x - 144$	Distributive Property
$= 6x^3 + 39x^2 + 24x - 144$	Simplify.

b. $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Division of functions
$= \frac{2x^2 + 5x - 12}{3x + 12}$	$f(x) = 2x^2 + 5x - 12$ and $g(x) = 3x + 12$
$= \frac{(2x - 3)(x + 4)}{3(x + 4)}$	Factor the numerator and denominator.
$= \frac{2x - 3}{3}, x \neq -4$	Divide numerator and denominator by $(x + 4)$.

The value –4 is included in the domain of $f(x)$ and the domain of $g(x)$, but it is excluded from the domain of $\left(\frac{f}{g}\right)(x)$.

Example 3 Evaluate Composition of Relations

If $f(x) = \{(2, 5), (4, –3), (–1, 6), (0, 3)\}$ and $g(x) = \{(–1, 2), (0, 4), (2, –1), (–5, 0)\}$, find $f \circ g$ and $g \circ f$.

To find $f \circ g$, evaluate $g(x)$ first. Then use the range of g as the domain of f and evaluate $f(x)$.

$f[g(–1)] = f(2)$ or 5	$g(–1) = 2$
$f[g(0)] = f(4)$ or –3	$g(0) = 4$
$f[g(2)] = f(–1)$ or 6	$g(2) = –1$
$f[g(–5)] = f(0)$ or 3	$g(–5) = 0$

$f \circ g = \{(–1, 5), (0, –3), (2, 6), (–5, 3)\}$

To find $g \circ f$, evaluate $f(x)$ first. Then use the range of f as the domain of g and evaluate $g(x)$.

$g[f(2)] = g(5)$	$g(5)$ is undefined.
$g[f(4)] = g(–3)$	$g(–3)$ is undefined.
$g[f(–1)] = g(6)$	$g(6)$ is undefined.
$g[f(0)] = g(3)$	$g(3)$ is undefined.

Since –3, 3, 5, and 6 are not in the domain of g, $g \circ f$ is undefined.

Example 4 Simplify Composition of Functions

a. Find $[f \circ g](x)$ and $[g \circ f](x)$ for $f(x) = 3x^2 + x – 1$ and $g(x) = –2x – 5$.

$[f \circ g](x) = f(g(x))$	Composition of functions
$= f(–2x – 5)$	Replace $g(x)$ with $–2x – 5$.
$= 3(–2x – 5)^2 + (–2x – 5) – 1$	Substitute $–2x – 5$ for x in $f(x)$.
$= 12x^2 + 58x + 69$	Simplify.

$[g \circ f](x) = g(f(x))$	Composition of functions
$= g(3x^2 + x – 1)$	Replace $f(x)$ with $3x^2 + x – 1$.
$= –2(3x^2 + x – 1) – 5$	Substitute $3x^2 + x – 1$ for x in $g(x)$.
$= –6x^2 – 2x – 3$	Simplify.

So, $[f \circ g](x) = 12x^2 – 62x + 69$ and $[g \circ f](x) = –6x^2 – 2x – 3$.

b. Evaluate $[f \circ g](x)$ and $[g \circ f](x)$ for $x = –5$.

$[f \circ g](x) = 12x^2 + 58x + 69$	Function from part *a*
$= 12(–5)^2 + 58(–5) + 69$	Replace x with –5.
$= 79$	Simplify.
$[g \circ f](x) = –6x^2 – 2x – 3$	Function from part *a*
$= –6(–5)^2 – 2(–5) – 3$	Replace x with –5.
$= –143$	Simplify.

So, $[f \circ g](–5) = 79$ and $[g \circ f](–5) = –143$.

Example 4 Use Composition of Functions

SHOPPING Sasha wants to buy a CD player that is on sale for 25% off the original price of $289. The sales tax on the player is 6.5%.

a. Express the price of the CD player after the discount and the price of the CD player after the sales tax using function notation. Let x represent the price of the CD player, $d(x)$ represent the price of the CD player after the 25% discount, and $t(x)$ represent the price after the sales tax.

Write equations for $d(x)$ and $t(x)$.

The CD player is discounted 25%, so the sale price is 100% – 25% = 75% of the original price.
$d(x) = 0.75x$

The sales tax rate is 6.5%.
$t(x) = x + 0.065x$

b. Find the total amount that Sasha will pay for the CD player.

When you buy an item in the store, the discount is applied to the price and then the sales tax is applied to that amount. So, find $(t \circ d)(x)$ or $t(d(x))$.

$t(d(x)) = t(0.75x)$	Replace $d(x)$ with $0.75x$.
$= 0.75x + 0.065(0.75x)$	Substitute $0.75x$ for x in $t(x)$.
$= 0.75x + 0.04875x$	Multiply.
$= 0.79875x$	Simplify.

Now, find the final price for the CD player with an original price of $289.

$t(d(x)) = 0.79875x$	Price function
$= 0.79875(289)$	Substitute 289 for x.
$= 230.83875$	Use a calculator.

Sasha will pay $230.84 for the CD player.