

Lesson 8–2

Example 1 Analyze the Equation of a Parabola

Write $y = -2x^2 - 4x + 3$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

$$y = -2x^2 - 4x + 3$$

$$y = -2(x^2 + 2x) + 3$$

$$y = -2(x^2 + 2x + \quad) + 3 - (-2)(\quad)$$

$$y = -2(x^2 + 2x + 1) + 3 - (-2)(1)$$

$$y = -2(x + 1)^2 + 5$$

$$y = -2[x - (-1)]^2 + 5$$

Original equation

Factor -2 from the x -terms.

Complete the square on the right side.

The 1 added when you complete the square is multiplied by -2 .

$(h, k) = (-1, 5)$

The vertex of this parabola is located at $(-1, 5)$, and the equation of the axis of symmetry is $x = -1$. The parabola opens downward.

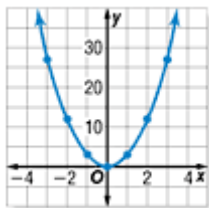
Example 2 Graph Parabolas

Graph each equation.

a. $y = 3x^2$

For this equation, $h = 0$ and $k = 0$. The vertex is at the origin. Since the equation of the axis of symmetry is $x = 0$, substitute some small positive integers for x and find the corresponding y -values.

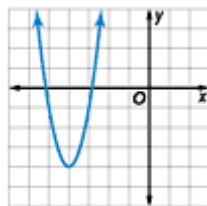
x	y
1	3
2	12
3	27



Since the graph is symmetric about the y -axis, the points at $(-1, 3)$, $(-2, 12)$, and $(-3, 27)$ are also on the parabola. Use all these points to draw the graph.

b. $y = 3(x + 4)^2 - 4$

Rewrite the equation as $y = 3[x - (-4)]^2 + (-4)$. Now, the equation is of the form $y = a(x - h)^2 + k$, where $h = -4$ and $k = -4$. The graph of this equation is the graph of $y = 3x^2$ in part a translated 4 units to the left and 4 units down. The vertex is now at $(-4, -4)$.



Example 3 Graph an Equation not in Standard Form

Graph $2x + y^2 = 8y - 6$.

First, write the equation in the form $x = a(y - k)^2 + h$.

$$2x + y^2 = 8y - 6$$

$$2x = -y^2 + 8y - 6$$

$$2x = -1(y^2 - 8y) - 6$$

$$2x = -1(y^2 - 8y + \quad) - 6 - (-1)(\quad)$$

$$2x = -1(y^2 - 8y + 16) - 6 - (-1)(16)$$

$$2x = -1(y - 4)^2 + 10$$

$$x = -\frac{1}{2}(y - 4)^2 + 5$$

There is a y^2 term, so isolate the y and y^2 terms.

Add $-y^2$ to each side.

Factor -1 from the y -terms.

Complete the square.

The 16 added when you complete the square is multiplied by -1 .

Divide each side by 2; $(h, k) = (5, 4)$

Then use the following information to draw the graph.

vertex: $(5, 4)$

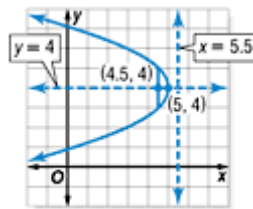
axis of symmetry: $y = 4$

$$\text{focus: } \left(5 + \frac{1}{4\left(-\frac{1}{2}\right)}, 4 \right) \text{ or } (4.5, 4)$$

$$\text{directrix: } x = 5 - \frac{1}{4\left(-\frac{1}{2}\right)} \text{ or } 5.5$$

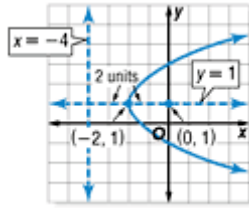
direction of opening: left, since $a < 0$

$$\text{length of latus rectum: } \left| \frac{1}{-\frac{1}{2}} \right| \text{ or } 2 \text{ units}$$



Example 4 Write and Graph an Equation for a Parabola
Write an equation for each parabola described. Then draw the graph.
a. vertex $(-2, 1)$, focus $(0, 1)$

Plot these two points on a coordinate plane. You can see that the parabola will have a horizontal axis of symmetry at $y = 1$. It will open to the right and the equation will then be of the form $x = a(y - k)^2 + h$. Since the focus is two units to the right of the vertex, the directrix is 2 units to the left of the vertex. The equation of the directrix is $x = -4$.



Let (x, y) be any point on this parabola. The distance from this point to the focus must be the same as the distance from this point to the directrix. The distance from a point to a line is measured along the perpendicular from the point to the line.

distance from (x, y) to $(0, 1)$ = distance from (x, y) to $(-4, y)$

$$\sqrt{(x - 0)^2 + (y - 1)^2} = \sqrt{[x - (-4)]^2 + (y - y)^2}$$

$$(x - 0)^2 + (y - 1)^2 = (x + 4)^2 + 0^2$$

$$x^2 + (y - 1)^2 = x^2 + 8x + 16$$

$$(y - 1)^2 - 16 = 8x$$

$$\frac{1}{8}(y - 1)^2 - 2 = x$$

Square each side.

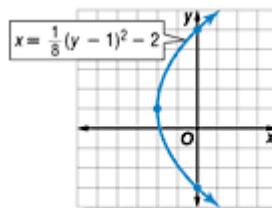
Square $x + 4$.

Isolate the x -term.

Divide each side by 8.

Graph the equation $x = \frac{1}{8}(y - 1)^2 - 2$ using the information found above and several more

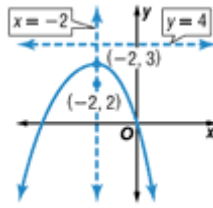
points. The length of the latus rectum is $\left| \frac{1}{1/8} \right|$ or



8 units, so the graph must pass through $(0, 5)$ and $(0, -3)$.

b. focus $(-2, 2)$, directrix $y = 4$

Plot the focus on a coordinate plane and draw the line for the directrix. You can see that the parabola will have a vertical axis of symmetry at $x = -2$. It will open downward, and the equation will then be of the form $y = a(x - h)^2 + k$. Since the focus and directrix are 2 units apart, the coordinates of the vertex are $(-2, 3)$.



In order to write an equation for the parabola, you must know the values for h , k , and a . You know that the vertex is at $(-2, 3)$, so $h = -2$ and $k = 3$. You also know that the focus is at $(-2, 2)$ and that the focus is at

$$\left(h, k + \frac{1}{4a} \right).$$
 Use these facts to

find the value of a .

$$2 = k + \frac{1}{4a}$$

$$2 = 3 + \frac{1}{4a}$$

Substitute 3 for k .

$$-1 = \frac{1}{4a}$$

Subtract 3 from each side.

$$-4a = 1$$

Multiply each side by $4a$.

$$a = -\frac{1}{4}$$

Divide each side by -4 .

Now use $h = -2$, $k = 3$, and $a = -\frac{1}{4}$ to write the equation of the parabola.

$$y = a(x - h)^2 + k$$

Equation for the parabola

$$y = -\frac{1}{4}(x + 2)^2 + 3$$

Substitute -2 for h , 3 for k , and $-\frac{1}{4}$ for a .

Graph the equation $y = -\frac{1}{4}(x + 2)^2 + 3$ using the information found above and several more

points. The length of the latus rectum is $\left| \frac{1}{-\frac{1}{4}} \right|$

or 4 units, so the graph must pass through $(0, 2)$ and $(-4, 2)$.

