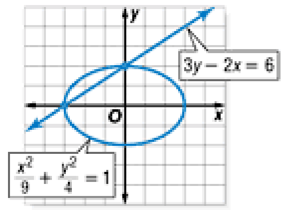
Lesson 8–7

Example 1 Linear–Quadratic System

Solve the system of equations.

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$
$$3y - 2x = 6$$



Since an ellipse is easy to graph, sketch this ellipse on grid paper. Rewrite $3y - 2x = 6$ as $y = \frac{2}{3}x + 2$ and graph the line on the same axes. The graph indicates that the ellipse and line intersect in two points. So the system has two solutions.

Use substitution to solve the system and find the exact coordinates of the solutions.

$\frac{x^2}{9} + \frac{y^2}{4} = 1$	First equation in the system
$\frac{x^2}{9} + \frac{\left(\frac{2}{3}x+2\right)^2}{4} = 1$	Substitute $\frac{2}{3}x + 2$ for y.
$4x^2 + 9\left(\frac{2}{3}x+2\right)^2 = 36$	Multiply each side by 36.
$4x^2 + 9\left(\frac{4}{9}x^2 + \frac{8}{3}x + 4\right) = 36$	Expand the binomial square.
$4x^2 + 4x^2 + 24x + 36 = 36$	Distributive Property
$8x^2 + 24x = 0$	Simplify and subtract 36 from each side.
$8x(x + 3) = 0$	Factor.
$8x = 0$ or $x + 3 = 0$	Zero Product Property
$x = 0$ $\quad$ $x = -3$	Solve each equation for x.

Now solve for y.

$y = \frac{2}{3}x + 2$	$y = \frac{2}{3}x + 2$	Equation for y in terms of x
$= \frac{2}{3}(0) + 2$	$= \frac{2}{3}(-3) + 2$	Substitute the x–values.
$= 2$	$= 0$	Simplify.

The solutions to the system are (0, 2) and (–3, 0). Based on the graph, these solutions are reasonable.

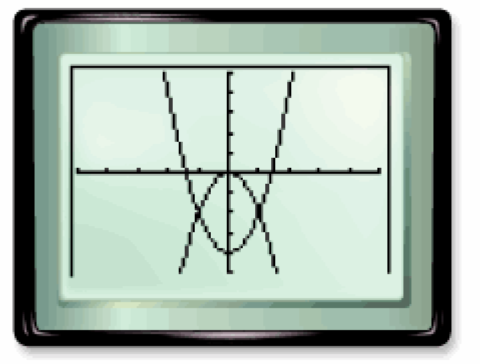
Example 2 **Quadratic–Quadratic System**

Solve the system of equations.

$y = -2x^2$

$y = 2x^2 - 4$

A graphing calculator indicates that the two parabolas intersect in two points. So, this system has two solutions.



Use the elimination method to solve the system.

Rewrite the equations.

$y = -2x^2 \quad \Rightarrow \quad y + 2x^2 = 0$

$y = 2x^2 - 4 \quad \Rightarrow \quad y - 2x^2 = -4$

$y + 2x^2 = 0$	First equation
$\underline{y - 2x^2 = -4}$	Second equation
$2y = -4$	Add.
$y = -2$	Divide each side by 2.

Now solve for *x*.

$y = -2x^2$	First equation
$-2 = -2x^2$	Substitute –2 for *y*.
$1 = x^2$	Divide each side by –2.
$\pm 1 = x$	Take the square root of each side.

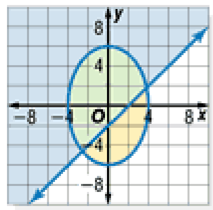
The solutions are (–1, –2) and (1, –2). Based on the graph, these solutions are reasonable.

Example 3 **System of Quadratic Inequalities**

Solve the system of inequalities by graphing.

$16y^2 + 36x^2 \leq 576$

$y \geq x - 2$



The graph of $16y^2 + 36x^2 \leq 576$ is the ellipse $16y^2 + 36x^2 = 576$. Write the equation in standard form.

$16y^2 + 36x^2 = 576$ Original equation

$\frac{y^2}{36} + \frac{x^2}{16} = 1$ Divide each side by 576.

Graph $\frac{y^2}{36} + \frac{x^2}{16} = 1$. The inequality $\frac{y^2}{36} + \frac{x^2}{16} \leq 1$ is the interior of the ellipse shaded in blue.

The graph of $y \geq x - 2$ is all points above the graph of $y = x - 2$. This region is shaded yellow.

The intersection of these two regions, shaded green, represents the solution of the system of inequalities.