

Lesson 9–6

Example 1 Solve a Rational Equation

Solve $\frac{29}{3x} - \frac{1}{3} = \frac{8}{x}$. Check your solution.

The LCD for the three denominators is $3x$.

$$\begin{aligned} \frac{29}{3x} - \frac{1}{3} &= \frac{8}{x} && \text{Original equation} \\ 3x\left(\frac{29}{3x} - \frac{1}{3}\right) &= 3x\left(\frac{8}{x}\right) && \text{Multiply each side by } 3x. \\ \cancel{3x}^1\left(\frac{29}{\cancel{3x}_1}\right) - \cancel{3}^1x\left(\frac{1}{\cancel{3}_1}\right) &= 3\cancel{x}^1\left(\frac{8}{\cancel{x}_1}\right) && \text{Distributive Property} \\ 29 - x &= 24 && \text{Simplify.} \\ -x &= -5 && \text{Subtract 29 from each side.} \\ x &= 5 && \text{Divide each side by } -1. \end{aligned}$$

Check

$$\begin{aligned} \frac{29}{3x} - \frac{1}{3} &= \frac{8}{x} && \text{Original equation} \\ \frac{29}{15} - \frac{1}{3} &= \frac{8}{5} && x = 5 \\ \frac{29}{15} - \frac{5}{15} &= \frac{8}{5} && \text{Simplify.} \\ \frac{8}{5} &= \frac{8}{5} && \checkmark \end{aligned}$$

Example 2 Elimination of a Possible Solution

Solve $\frac{2a^2}{a^2 - 25} - \frac{a}{a + 5} = \frac{7}{a - 5}$. Check your solution.

The LCD is $a^2 - 25$.

$$\begin{aligned} \frac{2a^2}{a^2 - 25} - \frac{a}{a + 5} &= \frac{7}{a - 5} && \text{Original equation} \\ (a^2 - 25)\left(\frac{2a^2}{a^2 - 25} - \frac{a}{a + 5}\right) &= (a^2 - 25)\left(\frac{7}{a - 5}\right) && \text{Multiply each side by } (a^2 - 25). \\ (a^2 - 25)\left(\frac{2a^2}{a^2 - 25}\right) - (a^2 - 25)\left(\frac{a}{a + 5}\right) &= (a^2 - 25)\left(\frac{7}{a - 5}\right) && \text{Distributive Property} \\ (2a^2) - (a - 5)(a) &= (a + 5)(7) && \text{Simplify.} \\ 2a^2 - a^2 + 5a &= 7a + 35 && \text{Simplify.} \\ a^2 - 2a - 35 &= 0 && \text{Subtract } (7a + 35) \text{ from each side.} \\ (a - 7)(a + 5) &= 0 && \text{Factor.} \\ a - 7 = 0 &\text{ or } &a + 5 = 0 && \text{Zero Product Property} \\ a = 7 &&& a = -5 \end{aligned}$$

Check:

$$\frac{2a^2}{a^2 - 25} - \frac{a}{a + 5} = \frac{7}{a - 5}$$

$$\frac{2(7)^2}{7^2 - 25} - \frac{7}{7 + 5} \stackrel{?}{=} \frac{7}{7 - 5}$$

$$\frac{98}{24} - \frac{7}{12} \stackrel{?}{=} \frac{7}{2}$$

$$\frac{7}{2} = \frac{7}{2} \quad \checkmark$$

The solution is 7.

Original equation

$$a = 7$$

Simplify.

Original equation

$$a = -5$$

Simplify.

$$\frac{2a^2}{a^2 - 25} - \frac{a}{a + 5} = \frac{7}{a - 5}$$

$$\frac{2(-5)^2}{(-5)^2 - 25} - \frac{-5}{-5 + 5} \stackrel{?}{=} \frac{7}{-5 - 5}$$

$$\frac{50}{0} - \frac{-5}{0} \stackrel{?}{=} \frac{7}{-10}$$

Since $a = -5$ results in a zero in the denominator, eliminate -5 from the list of solutions.

Example 3 Work Problem

CONSTRUCTION The Belgrade Intermediate School needs a new roof. Since the weather is turning cold quickly, the school board decides to hire two construction companies to complete the project. Rick's Roofing claims to be able to complete the job in 8.5 days. Phil's Fix-It Inc. claims to be able to complete the job in 11 days. If the time estimates of both companies are accurate, how long will it take for the two companies to complete the job working together?

In 1 day, Rick's Roofing can complete $\frac{1}{8.5}$ of the roof.

In 2 days, Rick's Roofing can complete $\frac{1}{8.5} \cdot 2$ or $\frac{2}{8.5}$ of the roof.

In d days, Rick's Roofing can complete $\frac{1}{8.5} \cdot d$ or $\frac{d}{8.5}$ of the roof.

Likewise, in d days, Phil's Fix-It Inc. can complete $\frac{1}{11} \cdot d$ or $\frac{d}{11}$ of the roof.

Together, they will complete the whole roof.

<i>Part completed</i>		<i>plus</i>	<i>part completed</i>		<i>equals</i>	<i>entire job.</i>
$\frac{d}{8.5}$	+		$\frac{d}{11}$	=		1

Solve the equation.

$$\frac{d}{8.5} + \frac{d}{11} = 1$$

$$93.5\left(\frac{d}{8.5} + \frac{d}{11}\right) = 93.5(1)$$

$$11d + 8.5d = 93.5$$

$$19.5d = 93.5$$

$$d \approx 4.8$$

Original equation

Multiply each side by 93.5.

Simplify.

Simplify.

Divide each side by 19.5.

It will take about 4.8 days to complete the roof.

Example 4 Rate Problem

TRAVEL A direct flight from Chicago, IL, to Washington, D.C., is approximately 600 miles. On a particular day, the speed of the wind is 50 miles per hour at the cruising altitude of a passenger jet. A jet leaves for this flight flying with the wind. Then the jet makes the return flight against the wind. The total in-air time for the jet for both flights was 2.7 hours. What is the speed of the jet in still air?

The formula that relates distance, time, and rate is $d = rt$ or $\frac{d}{r} = t$. Let r be the speed of the jet in still air.

Then the speed of the jet with the wind is $r + 50$, and the speed of the jet against the wind is $r - 50$.

<i>Time going with</i>		<i>time going against</i>			
<u>the wind</u>	<u>plus</u>	<u>the wind</u>	<u>equals</u>	<u>total time.</u>	
$\frac{600}{r + 50}$	+	$\frac{600}{r - 50}$	=	2.7	

Solve the equation.

$$\frac{600}{r + 50} + \frac{600}{r - 50} = 2.7 \quad \text{Original equation.}$$

$$10(r + 50)(r - 50)\left(\frac{600}{r + 50} + \frac{600}{r - 50}\right) = 10(r + 50)(r - 50)(2.7) \quad \text{Multiply each side by } 10(r + 50)(r - 50).$$

$$10(r - 50)(600) + 10(r + 50)(600) = 10(r + 50)(r - 50)(2.7) \quad \text{Distributive Property}$$

$$6000r - 300,000 + 6000r + 300,000 = 27r^2 - 67,500 \quad \text{Simplify.}$$

$$12,000r = 27r^2 - 67,500 \quad \text{Simplify.}$$

$$0 = 27r^2 - 12,000r - 67,500 \quad \text{Subtract } 12,000r \text{ from each side.}$$

$$0 = 3(9r^2 - 4,000r - 22,500) \quad \text{Factor out the GCF.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$r = \frac{-(-4,000) \pm \sqrt{(-4,000)^2 - 4(9)(-22,500)}}{2(9)} \quad x = r, a = 9, b = -4,000, \text{ and } c = -22,500$$

$$r = \frac{4,000 \pm \sqrt{16,810,000}}{18} \quad \text{Simplify.}$$

$$r = \frac{4000 \pm 4100}{18} \quad \text{Simplify.}$$

$$r = 450 \text{ or } -5.5 \quad \text{Simplify.}$$

Since the speed must be positive, the answer is 450 miles per hour.

Example 5 Solve a Rational Inequality

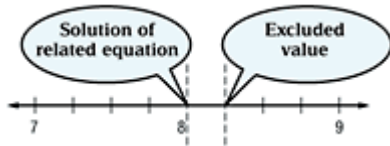
Solve $\frac{6}{b-8} < 24$.

Step 1 Values that make a denominator equal to 0 are excluded from the domain. For this inequality, the excluded value is 8.

Step 2 Solve the related equation.

$\frac{6}{b-8} = 24$	Related equation
$(b-8)\frac{6}{b-8} = (b-8)(24)$	Multiply each side by $(b-8)$.
$6 = 24b - 192$	Simplify.
$198 = 24b$	Add 192 to each side.
$8.25 = b$	Divide each side by 24.

Step 3 Draw vertical lines at the excluded value and at the solution to separate the number line into regions.



Now test a sample value in each region to determine if the values in the region satisfy the inequality.

Test $b = 0$.

$$\frac{6}{0-8} \stackrel{?}{<} 24$$
$$-\frac{3}{4} \stackrel{?}{<} 24$$
$$-\frac{3}{4} < 24 \quad \checkmark$$

$b < 8$ is a solution.

Test $b = 8.1$.

$$\frac{6}{8.1-8} \stackrel{?}{<} 24$$
$$60 \stackrel{?}{<} 24$$
$$60 \not< 24$$

$8 < b < 8.25$ is *not* a solution.

Test $b = 10$.

$$\frac{6}{10-8} \stackrel{?}{<} 24$$
$$3 \stackrel{?}{<} 24$$
$$3 < 24 \quad \checkmark$$

$b > 8.25$ is a solution.

The solution is $b < 8$ or $b > 8.25$.