Lesson 10–4

Example 1 Find Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

a. log 19 **KEYSTROKES:** [LOG] 19 [ENTER] about 1.2788

b. log 0.75 **KEYSTROKES:** [LOG] 0.75 [ENTER] about –0.1249

Example 2 Solve Logarithmic Equations Using Exponentiation

EARTHQUAKES In 1906, San Francisco experienced a major earthquake of magnitude 8.3. In 1989, another major quake hit the area with a magnitude of 7.1. The amount of energy *E*, in ergs, an earthquake releases is related to its Richter scale magnitude *M* by the equation log *E* = 11.8 + 1.5*M*. How much more energy did the 1906 quake release than the 1989 quake?

Find the amount of energy released by each quake and then subtract the two amounts.

1906 Earthquake

$\log E = 11.8 + 1.5M$	Write the formula.
$\log E = 11.8 + 1.5(8.3)$	Replace *M* with 8.3.
$\log E = 24.25$	Simplify.
$10^{\log E} = 10^{24.25}$	Write each side using exponents and base 10.
$E = 10^{24.25}$	Inverse Property of Exponents and Logarithms
$E \approx 1.78 \times 10^{24}$	Use a calculator.

1989 Earthquake

$\log E = 11.8 + 1.5M$	Write the formula.
$\log E = 11.8 + 1.5(7.1)$	Replace *M* with 7.1.
$\log E = 22.45$	Simplify.
$10^{\log E} = 10^{22.45}$	Write each side using exponents and base 10.
$E = 10^{22.45}$	Inverse Property of Exponents and Logarithms
$E \approx 2.82 \times 10^{22}$	Use a calculator.

energy of 1906 Earthquake – energy of 1989 Earthquake

$$1.78 \times 10^{24} - 2.82 \times 10^{22} = 178 \times 10^{22} - 2.82 \times 10^{22}$$
$$= (178 - 2.82)(10^{22})$$
$$= 175.18 \times 10^{22}$$
$$= 1.7518 \times 10^{24}$$

The 1906 Earthquake released 1.7518×10^{24} ergs more energy than the 1989 Earthquake.

Example 3 Solve Exponential Equations Using Logarithms

Solve $2^{a+3} = 34$.

$2^{a+3} = 34$	Original equation
$\log 2^{a+3} = \log 34$	Property of Equality for Logarithmic Functions
$(a + 3) \log 2 = \log 34$	Power Property of Logarithms
$a + 3 = \dfrac{\log 34}{\log 2}$	Divide each side by log 2.
$a = \dfrac{\log 34}{\log 2} - 3$	Subtract 3 from each side.
$a \approx \dfrac{1.5315}{0.3010} - 3$	Use a calculator.
$a \approx 2.0880$	Use a calculator.

The solution is approximately 2.0880.

CHECK You can check this answer using a calculator or by using estimation. Since $2^5 = 32$ and 32 is close to 34, the value of x is slightly greater than 2 since 2 + 3 = 5. Thus, 2.0880 is a reasonable solution.

Example 4 Solve Exponential Inequalities Using Logarithms

Solve $5^{b-1} \geq 10^{6-b}$.

$5^{b-1} \geq 10^{6-b}$	Original inequality
$\log 5^{b-1} \geq \log 10^{6-b}$	Property of Inequality for Logarithmic Functions
$(b - 1) \log 5 \geq (6 - b) \log 10$	Power Property of Logarithms
$b \log 5 - \log 5 \geq 6 \log 10 - b \log 10$	Distributive Property
$b \log 5 + b \log 10 \geq \log 5 + 6 \log 10$	Add b log 10 and log 5 to each side.
$b(\log 5 + \log 10) \geq \log 5 + 6 \log 10$	Distributive Property
$b \geq \dfrac{\log 5 + 6 \log 10}{\log 5 + \log 10}$	Divide each side by log 5 + log 10.
$b \geq \dfrac{0.6990 + 6(1)}{0.6990 + 1}$	Use a calculator.
$b \geq 3.9429$	Use a calculator.

The solution set is $\{b \mid b \geq 3.9429\}$.

CHECK Test $b = 4$.

$5^{b-1} \geq 10^{6-b}$	Original inequality
$5^{4-1} \overset{?}{\geq} 10^{6-4}$	Replace b with 4.
$5^3 \overset{?}{\geq} 10^2$	Simplify.
$125 \geq 100$ ✓	Simplify.

Example 5 Change of Base Formula

Express $\log_7 58$ in terms of common logarithms. Then approximate its value to four decimal places.

$$\log_7 58 = \frac{\log_{10} 58}{\log_{10} 7} \qquad \text{Change of Base Formula}$$

$$\approx 2.0867 \qquad \text{Use a calculator.}$$

The value of $\log_7 58$ is approximately 2.0867.