

Lesson 10–5

Example 1 Evaluate Natural Base Expressions

Use a calculator to evaluate each expression to four decimal places.

a. $e^{2.5}$ **KEYSTROKES:** $\boxed{2\text{nd}}$ $\boxed{[e^x]}$ 2.5 $\boxed{\text{ENTER}}$ about 12.1825

b. $e^{-4.6}$ **KEYSTROKES:** $\boxed{2\text{nd}}$ $\boxed{[e^x]}$ -4.6 $\boxed{\text{ENTER}}$ about 0.0101

Example 2 Evaluate Natural Logarithmic Expressions

Use a calculator to evaluate each expression to four decimal places.

a. $\ln 15$ **KEYSTROKES:** $\boxed{\text{LN}}$ 15 $\boxed{\text{ENTER}}$ about 2.7081

b. $\ln 0.75$ **KEYSTROKES:** $\boxed{\text{LN}}$ 0.75 $\boxed{\text{ENTER}}$ about -0.2877

Example 3 Write Equivalent Expressions

Write an equivalent exponential or logarithmic equation.

a. $e^{-x} = 2$ **b** $\ln x = 0.35$
 $e^{-x} = 2 \rightarrow \log_e 2 = -x$ $\ln x = 0.35 \rightarrow \log_e x = 0.35$
 $\ln 2 = -x$ $x = e^{0.35}$

Example 4 Inverse Property of Base e and Natural Logarithms

Evaluate each expression.

a. $e^{\ln 4.9}$
 $e^{\ln 4.9} = 4.9$

b. $\ln e^{2a-5}$
 $\ln e^{2a-5} = 2a - 5$

Example 5 Solve Base e Equations

Solve $-2e^{5x} + 10 = 6$.

$-2e^{5x} + 10 = 6$	Original equation
$-2e^{5x} = -4$	Subtract 10 from each side.
$e^{5x} = 2$	Divide each side by -2 .
$\ln e^{5x} = \ln 2$	Property of Equality for Logarithms
$5x = \ln 2$	Inverse Property of Exponents and Logarithms
$x = \frac{1}{5}(\ln 2)$	Multiply each side by $\frac{1}{5}$.
$x \approx 0.1386$	Use a calculator.

The solution is about 0.1386.

CHECK Substitute this value into the original equation.

$-2e^{5x} + 10 = 6$	Original equation
$-2e^{5(0.1386)} + 10 = 6$	Substitute 0.1386 for x .
$-3.9994 + 10 = 6$	Simplify.
$6.0006 \approx 6$ ✓	Simplify.

Example 6 Solve Base e InequalitiesSolve $6e^{-x} < 15$.

$6e^{-x} < 15$	Original inequality
$e^{-x} < 2.5$	Divide each side by 6.
$\ln e^{-x} < \ln 2.5$	Property of Inequalities for Logarithms
$-x < \ln 2.5$	Inverse Property of Exponents and Logarithms
$x > -\ln 2.5$	Divide each side by -1 and reverse the inequality.
$x > -0.9163$	Use a calculator.

The solution is about -0.9163 .**CHECK** Substitute $x = 0$ into the original inequality.

$$6e^{-x} < 15 \quad \text{Original inequality}$$

$$6e^0 < 15 \quad \text{Substitute 0 for } x.$$

$$6(1) < 15 \quad \text{Simplify.}$$

$$6 < 15 \quad \checkmark \quad \text{Simplify.}$$

Example 7 Solve Natural Log Equations and Inequalities

Solve each equation or inequality.

a. $\ln(6x - 3) + 3 = 10$

$\ln(6x - 3) + 3 = 10$	Original equation
$\ln(6x - 3) = 7$	Subtract 3 from each side.
$e^{\ln(6x - 3)} = e^7$	Write each side using exponents and base e .
$6x - 3 = e^7$	Inverse Property of Exponents and Logarithms
$6x = e^7 + 3$	Add 3 to each side.
$x = \frac{e^7 + 3}{6}$	Divide each side by 6.
$x \approx 183.2722$	Use a calculator.

The solution is about 183.2722. Check this solution using substitution or by graphing.

b. $\ln(3x + 2) < 5$

$\ln(3x + 2) < 5$	Original inequality
$e^{\ln(3x + 2)} < e^5$	Write each side using exponents and base e .
$3x + 2 < e^5$	Inverse Property of Exponents and Logarithms
$3x < e^5 - 2$	Subtract 2 from each side.
$x < \frac{e^5 - 2}{3}$	Divide each side by 3.
$x < 48.8044$	Use a calculator.

Recall that $(3x + 2)$ must be greater than 0 in order for $\ln(3x + 2)$ to be defined. Therefore, you must solve $3x + 2 > 0$.

$$\begin{array}{ll}
 3x + 2 > 0 & \text{Inequality to solve} \\
 3x > -2 & \text{Subtract 2 from each side.} \\
 x > -\frac{2}{3} & \text{Divide each side by 3.}
 \end{array}$$

The solution is $-\frac{2}{3} < x < 48.8044$.

CHECK Test values to the left and right of this interval and in the interval to check the solution.

Test $x = -0.75$.	Test $x = 0$.	Test $x = 50$.
$\ln(3x + 2) < 5$	$\ln(3x + 2) < 5$	$\ln(3x + 2) < 5$
$\ln[3(-0.75) + 2] \stackrel{?}{<} 5$	$\ln[3(0) + 2] \stackrel{?}{<} 5$	$\ln[3(50) + 2] \stackrel{?}{<} 5$
$\ln(-0.25) \stackrel{?}{<} 5$	$\ln(2) \stackrel{?}{<} 5$	$\ln(152) \stackrel{?}{<} 5$
$\ln(-0.25)$ is not defined.	$0.6931 < 5 \checkmark$	$5.0239 > 5$