

Lesson 11–2

Example 1 Find the Sum of an Arithmetic Series

Find the sum of $19 + 11 + 3 + (-5) + \dots + (-69)$.

You can see from the series that $a_1 = 19$, $a_n = -69$, and $d = -8$. You do not know n , the number of terms in the series. You need to write the entire series in order to find n . Use d to write the series.

$$19 + 11 + 3 + (-5) + (-13) + (-21) + (-29) + (-37) + (-45) + (-53) + (-61) + (-69)$$

You can see that $n = 12$.

Now, you can use either sum formula for this series.

Method 1

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{12} = \frac{12}{2}[19 + (-69)]$$

$$S_{12} = 6(-50)$$

$$S_{12} = -300$$

Sum formula

$n = 12$, $a_1 = 19$, $a_n = -69$, and
 $d = -8$

Simplify.

Multiply.

Method 2

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$S_{12} = \frac{12}{2}[2(19) + (12 - 1)(-8)]$$

$$S_{12} = 6(-50)$$

$$S_{12} = -300$$

The sum of the series is -300 .

Example 2 Find the First Term

RECREATION Miranda is planning to compete in a bicycle race. She rides her bicycle every day for sixteen consecutive days, increasing the distance of her ride by one-half mile each day. At the end of the sixteen days, she has ridden a total of 116 miles.

a. How many miles did she ride on the first day?

You know the values of n , S_n , and d . Use the sum formula that contains d .

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

Sum formula

$$S_{16} = \frac{16}{2}[2a_1 + (16 - 1)(0.5)]$$

$n = 16$, $d = 0.5$

$$116 = 8(2a_1 + 7.5)$$

$S_{16} = 116$

$$116 = 16a_1 + 60$$

Distributive Property

$$56 = 16a_1$$

Subtract 60 from each side.

$$3.5 = a_1$$

Divide each side by 16.

Miranda rode 3.5 miles on the first day.

Example 3 Find the First Three Terms

Find the first three terms of an arithmetic series in which $n = 12$, $a_n = -115$, and $S_n = -390$.

Step 1 Since you know n , a_n , and S_n , use

$$S_n = \frac{n}{2}(a_1 + a_n) \text{ to find } a_1.$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$-390 = \frac{12}{2}[a_1 + (-115)]$$

$$-390 = 6(a_1 - 115)$$

$$-390 = 6a_1 - 690$$

$$300 = 6a_1$$

$$50 = a_1$$

Step 2 Find d .

$$a_n = a_1 + (n - 1)d$$

$$-115 = 50 + (12 - 1)d$$

$$-165 = 11d$$

$$-15 = d$$

Step 3 Use d to determine a_2 and a_3 .

$$a_2 = 50 + (-15) \text{ or } 35$$

$$a_3 = 35 + (-15) \text{ or } 20$$

The first three terms are 50, 35, and 20.

Example 4 Evaluate a Sum in Sigma Notation

Evaluate $\sum_{i=1}^{20} (-6 - 7i)$.

You could list the 10 terms and then add. However, since there are 20 terms, it is quicker to use either sum formula.

First find a_1 and d , since you know $n = 20$.

$$a_1 = -6 - 7(1) \\ = -13$$

→

$$a_2 = -6 - 7(2) \\ = -20$$

→

$$d = -20 - (-13) = -7$$

Now use these values in the sum formula.

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

Sum formula

$$S_n = \frac{20}{2}[2(-13) + (20 - 1)(-7)]$$

$n = 20$, $a_1 = -13$, $d = -7$

$$= 10(-159)$$

Simplify.

$$= -1590$$

Multiply.

The sum of the series is -1590 .