

Lesson 11-5

Example 1 Sum of an Infinite Geometric Series

Find the sum of each infinite geometric series, if it exists.

a. $16 - 22.4 - 31.36 + \dots$

First, find the value of r to determine if the sum exists.

$$a_1 = 16 \text{ and } a_2 = -22.4, \text{ so } r = \frac{-22.4}{16} \text{ or } -1.4. \text{ Since } |-1.4| \geq 1, \text{ the sum does not exist.}$$

b. $-5 + 1.25 - 0.3125 + \dots$

First, find the value of r to determine if the sum exists.

$$a_1 = -5 \text{ and } a_2 = 1.25, \text{ so } r = \frac{1.25}{-5} \text{ or } -0.25. \text{ Since } |-0.25| < 1, \text{ the sum exists. Now use the formula for the sum of an infinite geometric series.}$$

$$\begin{aligned} S &= \frac{a_1}{1 - r} && \text{Sum formula} \\ &= \frac{-5}{1 - (-0.25)} && a_1 = -5, r = -0.25 \\ &= \frac{-5}{1.25} \text{ or } -4 && \text{Simplify.} \end{aligned}$$

The sum of the series is -4 .

Example 2 Infinite Series in Sigma Notation

Evaluate $\sum_{n=1}^{\infty} -\frac{2}{5} \left(\frac{1}{2}\right)^{n-1}$.

In this infinite geometric series, $a_1 = -\frac{2}{5}$ and $r = \frac{1}{2}$.

$$\begin{aligned} S &= \frac{a_1}{1 - r} && \text{Sum formula} \\ &= \frac{-\frac{2}{5}}{1 - \frac{1}{2}} && a_1 = -\frac{2}{5}, r = \frac{1}{2} \\ &= \frac{-\frac{2}{5}}{\frac{1}{2}} \text{ or } -\frac{4}{5} && \text{Simplify.} \end{aligned}$$

$$\text{Thus, } \sum_{n=1}^{\infty} -\frac{2}{5} \left(\frac{1}{2}\right)^{n-1} = -\frac{4}{5}.$$

Example 3 Write a Repeating Decimal as a Fraction

Write $0.\overline{587}$ as a fraction.

Method 1

Write the repeating decimal as a sum.

$$\begin{aligned}0.\overline{587} &= 0.587587587 \dots \\ &= 0.587 + 0.000587 + 0.000000587 + \dots \\ &= \frac{587}{1000} + \frac{587}{1,000,000} + \frac{587}{1,000,000,000} + \dots\end{aligned}$$

In this series, $a_1 = \frac{587}{1000}$ and $r = \frac{1}{1000}$.

$$\begin{aligned}S &= \frac{a_1}{1 - r} && \text{Sum formula} \\ &= \frac{\frac{587}{1000}}{1 - \frac{1}{1000}} && a_1 = \frac{587}{1000}, r = \frac{1}{1000} \\ &= \frac{587}{999} && \text{Subtract.} \\ &= \frac{587}{999} && \text{Simplify.}\end{aligned}$$

$$\text{Thus, } 0.\overline{587} = \frac{587}{999}.$$

Method 2

$$S = 0.\overline{587}$$

$$S = 0.587587587 \dots$$

$$1000S = 587.587587 \dots$$

$$999S = 587$$

$$S = \frac{587}{999}$$

Label the given decimal.

Repeating decimal

Multiply each side by 1000.

Subtract the second equation from the third.

Divide each side by

999.