**Lesson 3–4**

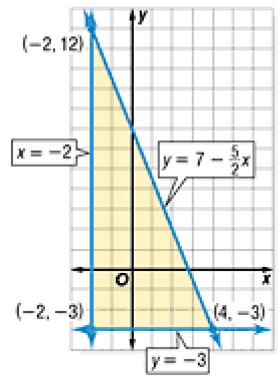
## Example 1 **Bounded Region**

**Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 2x - 5y$ for this region.**

$y \geq -3$

$x \geq -2$

$y \leq 7 - \frac{5}{2}x$




**Step 1** Find the vertices of the region. Graph the inequalities.

The polygon formed is a triangle with vertices at (–2, 12), (–2, –3), and (4, –3).

**Step 2** Use a table to find the maximum and minimum values for $f(x, y)$. Substitute the coordinates of the vertices into the function.

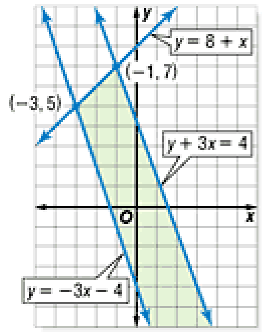
| $(x, y)$ | $2x - 5y$ | $f(x, y)$ | |
|---|---|---|---|
| (–2, 12) | 2(–2) – 5(12) | –64 | ← minimum |
| (–2, –3) | 2(–2) – 5(–3) | 11 | |
| (4, –3) | 2(4) – 5(–3) | 23 | ← maximum |

The maximum value is 23 at (4, –3). The minimum value is –64 at (–2, 12).

## Example 2 Unbounded Region

**Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 4y - 3x$ for this region.**

**$y + 3x \leq 4$**
**$y \geq -3x - 4$**
**$y \leq 8 + x$**




Graph the system of inequalities. There are only two points of intersection, (–1, 7) and (–3, 5).

| $(x, y)$ | $4y - 3x$ | $f(x, y)$ |
|---|---|---|
| (–1, 7) | 4(7) – 3(–1) | 31 |
| (–3, 5) | 4(5) – 3(–3) | 29 |

The maximum is 31 at (–1, 7).

Although $f(-3, 5)$ is 29, it is not the minimum value since there are other points in the solution that produce lesser values. For example, $f(0, 2) = 8$ and $f(-1, -1) = -1$. It appears that because the region is unbounded, $f(x, y)$ has no minimum value.

## Example 3 **Use Linear Programming**

**BUSINESS Ingrid is planning to start a home–based business. She will be baking decorated cakes and specialty pies. She estimates that a decorated cake will take 75 minutes to prepare and a specialty pie will take 30 minutes to prepare. She plans to work no more than 40 hours per week and does not want to make more than 60 pies in any one week. If she plans to charge $34 for a cake and $16 for a pie, find a combination of cakes and pies that will maximize her income for a week.**

**Step 1** Define the variables.
$c$ = the number of cakes
$p$ = the number of pies

**Step 2** Write a system of inequalities.
Since the number of baked items cannot be negative, $c$ and $p$ must be nonnegative numbers.

$c \geq 0$
$p \geq 0$

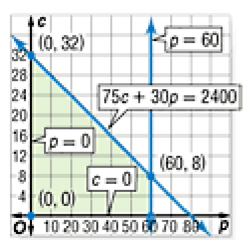
A cake takes 75 minutes, and a pie takes 30 minutes to prepare. There are 40 hours available per week.

$75c + 30p \leq 2400$ 40 hours = 2400 minutes

Ingrid does not want to make more than 60 pies in one week.

$p \leq 60$

**Step 3** Graph the system of inequalities.




**Step 4** Find the coordinates of the vertices of the feasible region. From the graph, the vertices of the feasible region are at (0, 0), (0, 32), (60, 8), and (60, 0).

**Step 5** Write a function to be maximized or minimized. The function that describes the income is $f(p, c) = 16p + 34c$.

**Step 6** Substitute the coordinates of the vertices into the function.

| $(p, c)$ | $16p + 34c$ | $f(p, c)$ |
|---|---|---|
| (0, 0) | 16(0) + 34(0) | 0 |
| (0, 32) | 16(0) + 34(32) | 1088 |
| (60, 8) | 16(60) + 34(8) | 1232 |
| (60, 0) | 16(60) + 34(0) | 960 |

**Step 7** Select the greatest or least result. Answer the problem.

The maximum value of the function is 1232 at (60, 8). This means that the maximum income is $1232 when Ingrid makes 60 pies and 8 cakes per week.