Lesson 4–6

Example 1 System of Two Equations

Use Cramer's Rule to solve the system of equations.

$4x + 8y = -4$

$-7x + 2y = 11$

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad \text{Cramer's Rule} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} -4 & 8 \\ 11 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & 8 \\ -7 & 2 \end{vmatrix}} \qquad a = 4,\ b = 8,\ c = -7,\ d = 2,\ e = -4, \text{ and } f = 11 \qquad = \frac{\begin{vmatrix} 4 & -4 \\ -7 & 11 \end{vmatrix}}{\begin{vmatrix} 4 & 8 \\ -7 & 2 \end{vmatrix}}$$

$$= \frac{-4(2) - 8(11)}{4(2) - 8(-7)} \qquad \text{Evaluate each determinant.} \qquad = \frac{4(11) - (-4)(-7)}{4(2) - 8(-7)}$$

$$= \frac{-96}{64} \text{ or } -1.5 \qquad \text{Multiply.} \qquad = \frac{16}{64} \text{ or } 0.25$$

The solution is (–1.5, 0.25).

Example 2 Use Cramer's Rule

MONEY On March 31, 2000, the number of $500 bills in circulation in the U.S. was the same as two times the number of $1000 bills minus 46,648. The total amount of money from $500 and $1000 bills was $310,204,000. Source: Kids Almanac

a. Write a system of equations that can be used to find the number of $500 bills and the number of $1000 bills in circulation on March 31, 2000.

Let f represent the number of $500 bills in circulation.
Let t represent the number of $1000 bills in circulation.

$f = 2t - 46{,}648$	The number of $500 bills is 2 times number of $1000 bills minus 46,648.
$500f + 1000t = 310{,}204{,}000$	500 times number of $500 bills plus 1000 times number of $1000 bills totals 310,204,000.

b. Find the number of each type of bill that was in circulation on March 31, 2000.

Use Cramer's Rule to solve this system of equations. First, rewrite the first equation in standard form.

$f - 2t = -46{,}648$	First equation
$500f + 1000t = 310{,}204{,}000$	Second equation

$$f = \frac{\begin{vmatrix} -46,648 & -2 \\ 310,204,000 & 1000 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 500 & 1000 \end{vmatrix}} \qquad t = \frac{\begin{vmatrix} 1 & -46,648 \\ 500 & 310,204,000 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 500 & 1000 \end{vmatrix}}$$

$$= \frac{-46,648(1000) - (-2)(310,204,000)}{1(1000) - (-2)(500)} \qquad = \frac{1(310,204,000) - (-46,648)(500)}{1(1000) - (-2)(500)}$$

$$= \frac{-46,648,000 + 620,408,000}{1000 + 1000} \qquad = \frac{310,204,000 + 23,324,000}{1000 + 1000}$$

$$= 286,880 \qquad = 166,764$$

The solution of the system is (286,880, 166,764).

So, there were 286,880 \$500 bills in circulation and 166,764 \$1000 bills in circulation.

Example 3 System of Three Equations

Use Cramer's Rule to solve the system of equations.

$$2x + 3y - 4z = -7$$
$$x + y + z = 3$$
$$-4x - y + 2z = 9$$

$$x = \frac{\begin{vmatrix} -7 & 3 & -4 \\ 3 & 1 & 1 \\ 9 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & -4 \\ 1 & 1 & 1 \\ -4 & -1 & 2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 2 & -7 & -4 \\ 1 & 3 & 1 \\ -4 & 9 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & -4 \\ 1 & 1 & 1 \\ -4 & -1 & 2 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} 2 & 3 & -7 \\ 1 & 1 & 3 \\ -4 & -1 & 9 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & -4 \\ 1 & 1 & 1 \\ -4 & -1 & 2 \end{vmatrix}}$$

Use a calculator to evaluate each determinant.

$x = \frac{36}{-24}$ or -1.5 $\qquad y = \frac{-48}{-24}$ or 2 $\qquad z = \frac{-60}{-24}$ or 2.5

The solution is (–1.5, 2, 2.5).