Lesson 4–8

Example 1 Two–Variable Matrix Equation

Write a matrix equation for the system of equations.
$-4a + 7b = 17$
$a - 9b = -113$

Determine the coefficient, variable, and constant matrices.

$$\begin{aligned} -4a + 7b &= 17 \\ a - 9b &= -113 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} -4 & 7 \\ 1 & -9 \end{bmatrix} \quad \begin{bmatrix} a \\ b \end{bmatrix} \quad \begin{bmatrix} 17 \\ -113 \end{bmatrix}$$

Write the matrix equation.

$$A \cdot X = B$$

$$\begin{bmatrix} -4 & 7 \\ 1 & -9 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 17 \\ -113 \end{bmatrix}$$

Example 2 Solve a Problem Using a Matrix Equation

FOOTBALL Mani Peters is the kicker for the Winston High School football team. In one game, Mani, kicked the ball 9 times for a total of 13 points. Two of his kicks were no good. He kicked some field goals worth 3 points each and some points after touchdowns worth 1 point each.

a. Write a system of equations that represent the situation.

Let f represent the number of field goals Mani kicked.
Let p represent the number of points after touchdowns he kicked.

Since Mani kicked the ball 9 times and 2 of the kicks were no good, the number of field goals plus the number of points after touchdowns equals 9 - 2 or 7.

$f + p = 7$

Since Mani scored 13 points, three times the number of field goals plus the number of points after touchdowns equals 13.

$3f + p = 13$

b. Write the matrix for the system of equations.

Determine the coefficient, variable, and constant matrices.

$$\begin{aligned} f + p &= 7 \\ 3f + p &= 13 \end{aligned} \quad \rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \quad \begin{bmatrix} f \\ p \end{bmatrix} \quad \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

Write the matrix equation.

$$\begin{matrix} A & \cdot & X & = & B \end{matrix}$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} f \\ p \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

Example 3 **Solve Matrix Equations**

Solve $\begin{bmatrix} \mathbf{3} & \mathbf{-7} \\ \mathbf{-1} & \mathbf{9} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{m} \\ \boldsymbol{p} \end{bmatrix} = \begin{bmatrix} \mathbf{-29} \\ \mathbf{23} \end{bmatrix}$**.**

In the matrix equation $A = \begin{bmatrix} 3 & -7 \\ -1 & 9 \end{bmatrix}$, $X = \begin{bmatrix} -29 \\ 23 \end{bmatrix}$, and $B = \begin{bmatrix} -29 \\ 23 \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{27 - 7}\begin{bmatrix} 9 & 7 \\ 1 & 3 \end{bmatrix} \text{ or } \frac{1}{20}\begin{bmatrix} 9 & 7 \\ 1 & 3 \end{bmatrix}$$

Step 2 Multiply each side of the matrix equation by the inverse matrix.

$$\frac{1}{20}\begin{bmatrix} 9 & 7 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -7 \\ -1 & 9 \end{bmatrix} \cdot \begin{bmatrix} m \\ p \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 9 & 7 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} -29 \\ 23 \end{bmatrix}$$ Multiply each side by A^{-1}.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ p \end{bmatrix} = \frac{1}{20}\begin{bmatrix} -100 \\ 40 \end{bmatrix}$$ Multiply matrices.

$$\begin{bmatrix} m \\ p \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

The solution is (–5, 2). Check this solution in the original equations.

Example 4 **Solve Matrix Equations**

Solve $\begin{bmatrix} \mathbf{-3} & \mathbf{4} \\ \mathbf{6} & \mathbf{-8} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \mathbf{15} \\ \mathbf{10} \end{bmatrix}$**.**

Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{24 - 24}\begin{bmatrix} -8 & -4 \\ -6 & -3 \end{bmatrix}$$

The determinant of the coefficient matrix $\begin{bmatrix} -3 & 4 \\ 6 & -8 \end{bmatrix}$ is zero, so A^{-1} does not exist. There is no unique solution of this system.

The matrix equation represents the system of equations below.

$-3x + 4y = 15$

$6x - 8y = 10$

Graph the system of equations Since the lines are parallel, this system has no solution. Therefore the system is inconsistent.