

## Lesson 5-3

### Example 1 Divide a Polynomial by a Monomial

Simplify  $\frac{25a^5b^3 - 20a^4b^2 + 10a^3b}{5a^2b}$ .

$$\begin{aligned} \frac{25a^5b^3 - 20a^4b^2 + 10a^3b}{5a^2b} &= \frac{25a^5b^3}{5a^2b} - \frac{20a^4b^2}{5a^2b} + \frac{10a^3b}{5a^2b} && \text{Sum of quotients.} \\ &= \frac{25}{5} \cdot a^{5-2} b^{3-1} - \frac{20}{5} \cdot a^{4-2} b^{2-1} + \frac{10}{5} \cdot a^{3-2} b^{1-1} && \text{Divide.} \\ &= 5a^3b^2 - 4a^2b + 2a && b^{1-1} = b^0 \text{ or } 1 \end{aligned}$$

### Example 2 Division Algorithm

Use long division to find  $(x^2 - 7x - 18) \div (x + 2)$

$$\begin{array}{r} x \\ x+2 \overline{)x^2 - 7x - 18} \\ \underline{(-) x^2 + 2x} \phantom{- 18} \\ -9x - 18 \\ \underline{-7x - (2x) = -9x} \\ -9x - 18 \\ \underline{-9x - 18} \\ 0 \end{array} \quad \Rightarrow \quad \begin{array}{r} x - 9 \\ x+2 \overline{)x^2 - 7x - 18} \\ \underline{(-) x^2 + 2x} \\ -9x - 18 \\ \underline{-9x - 18} \\ 0 \end{array}$$

The quotient is  $x - 9$ . The remainder is 0.

### Example 3 Quotient with Remainder

Multiple-Choice Test Item

Which expression is equal to  $(m^2 + m - 35)(m - 5)^{-1}$ ?

A.  $m + 6 - \frac{5}{m-5}$

B.  $m + 6$

C.  $m - 6$

D.  $m - 6 - \frac{5}{m-5}$

#### Read the Test Item

Since the second factor has an exponent of  $-1$ , this is a division problem.

$$(m^2 + m - 35)(m - 5)^{-1} = \frac{m^2 + m - 35}{m - 5}$$

#### Solve the Test Item

$$\begin{array}{r} m + 6 \\ m-5 \overline{)m^2 + m - 35} \\ \underline{(-) m^2 - 5m} \\ 6m - 35 \\ \underline{(-) 6m - 30} \\ -5 \end{array} \quad \begin{array}{l} m(m-5) = m^2 - 5m \\ m - (-5m) = 6m \\ 6(m-5) = 6m - 30 \\ \text{Subtract. } -35 - (-30) = -5 \end{array}$$



**Example 5 Divisor with First Coefficient Other than 1**  
 Use synthetic division to find  $(12x^3 - 7x^2 + 4x - 3) \div (3x - 1)$ .

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{12x^3 - 7x^2 + 4x - 3}{3x - 1} = \frac{(12x^3 - 7x^2 + 4x - 3) \div 3}{(3x - 1) \div 3}$$

Divide the numerator and denominator by 3.

$$= \frac{4x^3 - \frac{7}{3}x^2 + \frac{4}{3}x - 1}{x - \frac{1}{3}}$$

Simplify numerator and denominator.

$$x - r = x - \frac{1}{3}, \text{ so } r = \frac{1}{3}.$$

$$\begin{array}{r|rrrr} \frac{1}{3} & 4 & -\frac{7}{3} & \frac{4}{3} & -1 \\ & & \frac{4}{3} & -\frac{1}{3} & \frac{1}{3} \\ \hline & 4 & -1 & 1 & -\frac{2}{3} \end{array}$$

The result is  $4x^2 - x + 1 - \frac{\frac{2}{3}}{x - \frac{1}{3}}$ . Now simplify the fraction.

$$\frac{\frac{2}{3}}{x - \frac{1}{3}} = \frac{2}{3} \div \left(x - \frac{1}{3}\right)$$

Rewrite as a division expression.

$$= \frac{2}{3} \div \frac{3x-1}{3}$$

$$x - \frac{1}{3} = \frac{3x}{3} - \frac{1}{3} = \frac{3x-1}{3}$$

$$= \frac{2}{3} \cdot \frac{3}{3x-1}$$

Multiply by the reciprocal.

$$= \frac{2}{3x-1}$$

Multiply.

The solution is  $4x^2 - x + 1 - \frac{2}{3x - 1}$ .

**Check** Divide using long division.

$$\begin{array}{r} 4x^2 - x + 1 \\ 3x-1 \overline{) 12x^3 - 7x^2 + 4x - 3} \\ \underline{(-) 12x^3 - 4x^2} \phantom{- 3} \\ -3x^2 + 4x \phantom{- 3} \\ \underline{(-) -3x^2 + x} \phantom{- 3} \\ 3x - 3 \phantom{- 3} \\ \underline{(-) 3x - 1} \\ -2 \end{array}$$

The result is  $4x^2 - x + 1 - \frac{2}{3x - 1}$ . ✓