

Lesson 5-4

Example 1 GCF

Factor $-18ax^3 - 12bx^2 + 6x^2$.

$$\begin{aligned} -18ax^3 - 12bx^2 + 6x^2 &= (-1 \cdot 2 \cdot 3 \cdot 3 \cdot a \cdot x \cdot x \cdot x) + (-1 \cdot 2 \cdot 2 \cdot 3 \cdot b \cdot x \cdot x) + (2 \cdot 3 \cdot x \cdot x) \\ &= (6x^2 \cdot -3ax) + (6x^2 \cdot -2b) + (6x^2 \cdot 1) \\ &= 6x^2(-3ax - 2b + 1) \end{aligned}$$

The GCF is $6x^2$.

The remaining polynomial cannot be factored.

Check this result by finding the product.

Example 2 Grouping

Factor $x^3 + 5x^2 - 7x - 35$.

$$\begin{aligned} x^3 + 5x^2 - 7x - 35 &= (x^3 + 5x^2) + (-7x - 35) \\ &= x^2(x + 5) + (-7)(x + 5) \\ &= (x + 5)(x^2 - 7) \end{aligned}$$

Group to find a GCF.

Factor the GCF of each polynomial.

Distributive Property

Example 3 Two or Three Terms

Factor each polynomial.

a. $6a^2 - 13a - 5$

To find the coefficients of the a -terms, you must find two numbers whose product is $6(-5)$ or -30 , and whose sum is -13 . The coefficients must be -15 and 2 since $-15(2) = -30$ and $-15 + 2 = -13$. Rewrite the expression using $-15a$ and $2a$ and factor by grouping.

$$\begin{aligned} 6a^2 - 13a - 5 &= 6a^2 - 15a + 2a - 5 && \text{Substitute } -15a + 2a \text{ for } -13a. \\ &= (6a^2 - 15a) + (2a - 5) && \text{Associative Property} \\ &= 3a(2a - 5) + 1(2a - 5) && \text{Factor out the GCF of each group.} \\ &= (3a + 1)(2a - 5) && \text{Distributive Property} \end{aligned}$$

b. $20c^2 - 20cd + 5d^2$

$$\begin{aligned} 20c^2 - 20cd + 5d^2 &= 5(4c^2 - 4cd + d^2) && \text{Factor out the GCF.} \\ &= 5(2c - d)^2 && \text{Perfect square trinomial} \end{aligned}$$

c. $z^3 - 8x^3$

$z^3 = (z)^3$ and $(2x)^3 = 8x^3$. Thus, this is the difference of two cubes.

$$\begin{aligned} z^3 - 8x^3 &= (z - 2x)[z^2 + z(2x) + (2x)^2] && \text{Difference of two cubes formula with } a = z \text{ and } b = 2x \\ &= (z - 2x)(z^2 + 2zx + 4x^2) && \text{Simplify.} \end{aligned}$$

d. $z^6 - 64$

$$\begin{aligned} z^6 - 64 &= (z^3 + 8)(z^3 - 8) && \text{Difference of two squares} \\ &= (z + 2)(z^2 - 2z + 4)(z - 2)(z + 2z + 4) && \text{Sum and difference of two cubes} \end{aligned}$$

Example 4 Quotient of Two Trinomials

Simplify $\frac{x^2 + 7x - 30}{x^2 + 15x + 50}$.

$$\frac{x^2 + 7x - 30}{x^2 + 15x + 50} = \frac{(x-3) \overset{1}{\cancel{(x+10)}}}{(x+5) \overset{1}{\cancel{(x+10)}}}$$

Factor the numerator and denominator.

$$= \frac{x-3}{x+5}$$

Divide. Assume $x \neq -10, -5$.

Therefore, $\frac{x^2 + 7x - 30}{x^2 + 15x + 50} = \frac{x-3}{x+5}$, if $x \neq -10, -5$.