Lesson 5–6

Example 1 Square Root of a Product

Simplify $\sqrt{50a^5x^4}$.

$\sqrt{50a^5x^4} = \sqrt{2 \cdot 5^2 \cdot (a^2)^2 \cdot a \cdot (x^2)^2}$	Factor into squares where possible.
$= \sqrt{2} \cdot \sqrt{5^2} \cdot \sqrt{(a^2)^2} \cdot \sqrt{a} \cdot \sqrt{(x^2)^2}$	Product Property of Radicals
$= 5a^2x^2\sqrt{2a}$	Simplify.

Example 2 Simplify Quotients

Simplify each expression.

a. $\sqrt{\dfrac{d^{12}}{n^9}}$

$\sqrt{\dfrac{d^{12}}{n^9}} = \dfrac{\sqrt{d^{12}}}{\sqrt{n^9}}$	Quotient Property
$= \dfrac{\sqrt{(d^6)^2}}{\sqrt{(n^4)^2 \cdot n}}$	Factor into squares.
$= \dfrac{\sqrt{(d^6)^2}}{\sqrt{(n^4)^2} \cdot \sqrt{n}}$	Product Property
$= \dfrac{d^6}{n^4\sqrt{n}}$	$\sqrt{(d^6)^2} = d^6$, $\sqrt{(n^4)^2} = n^4$
$= \dfrac{d^6}{n^4\sqrt{n}} \cdot \dfrac{\sqrt{n}}{\sqrt{n}}$	Rationalize the denominator.
$= \dfrac{d^6\sqrt{n}}{n^5}$	$\sqrt{n} \cdot \sqrt{n} = n$

b. $\sqrt[3]{\dfrac{9}{2m}}$

$\sqrt[3]{\dfrac{9}{2m}} = \dfrac{\sqrt[3]{9}}{\sqrt[3]{2m}}$	Quotient Property
$= \dfrac{\sqrt[3]{9}}{\sqrt[3]{2m}} \cdot \dfrac{\sqrt[3]{4m^2}}{\sqrt[3]{4m^2}}$	Rationalize the denominator.
$= \dfrac{\sqrt[3]{9 \cdot 4m^2}}{\sqrt[3]{2m \cdot 4m^2}}$	Product Property
$= \dfrac{\sqrt[3]{36m^2}}{\sqrt[3]{8m^3}}$	Multiply.
$= \dfrac{\sqrt[3]{36m^2}}{2m}$	$\sqrt[3]{8m^3} = 2m$

Example 3 Multiply Radicals

Simplify $2\sqrt{18a^3} \cdot 5\sqrt{72a^2b^3}$.

$2\sqrt{18a^3} \cdot 5\sqrt{72a^2b^3} = 2 \cdot 5 \cdot \sqrt{18a^3 \cdot 72a^2b^3}$	Product Property of Radicals
$= 10 \cdot \sqrt{(36)^2 \cdot (a^2)^2 \cdot a \cdot b^2 \cdot b}$	Factor into squares where possible.
$= 10 \cdot \sqrt{36^2} \cdot \sqrt{(a^2)^2} \cdot \sqrt{a} \cdot \sqrt{b^2} \cdot \sqrt{b}$	Product Property of Radicals
$= 10 \cdot 36 \cdot a^2 \cdot b \cdot \sqrt{a} \cdot \sqrt{b}$ or $360a^2b\sqrt{ab}$	Multiply.

Example 4 Add and Subtract Radicals

Simplify $-4\sqrt{18} + 5\sqrt{50} - 2\sqrt{98}$

$-4\sqrt{18} + 5\sqrt{50} - 2\sqrt{98}$

$= 2\sqrt{2 \cdot 3^2} + 5\sqrt{2 \cdot 5^2} - 2\sqrt{2 \cdot 7^2}$ Factor using squares.

$= 2\sqrt{2} \cdot \sqrt{3^2} + 5\sqrt{2} \cdot \sqrt{5^2} - 2\sqrt{2} \cdot \sqrt{7^2}$ Product Property

$= 2 \cdot 3 \cdot \sqrt{2} + 5 \cdot 5 \cdot \sqrt{2} - 2 \cdot 7 \cdot \sqrt{2}$ $\sqrt{3^2} = 3$, $\sqrt{5^2} = 5$, $\sqrt{7^2} = 7$

$= 6\sqrt{2} + 25\sqrt{2} - 14\sqrt{2}$ Multiply.

$= 17\sqrt{2}$ Combine like radicals.

Example 5 Multiply Radicals

a. $(2\sqrt{2} - \sqrt{7})(3\sqrt{7} - 5)$

$$(2\sqrt{2} - \sqrt{7})(3\sqrt{7} - 5) = \overset{F}{2\sqrt{2} \cdot 3\sqrt{7}} + \overset{O}{2\sqrt{2} \cdot (-5)} + \overset{I}{(-\sqrt{7}) \cdot (3\sqrt{7})} + \overset{L}{(-\sqrt{7}) \cdot (-5)}$$

$= 6\sqrt{14} - 10\sqrt{2} - 3\sqrt{7^2} + 5\sqrt{7}$ Product Property

$= 6\sqrt{14} - 10\sqrt{2} - 21 + 5\sqrt{7}$ $3\sqrt{7^2} = 3 \cdot 7 = 21$

b. $(4\sqrt{5} - 3\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$

$(4\sqrt{5} - 3\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$

$= 4\sqrt{5} \cdot 4\sqrt{5} + 4\sqrt{5}(3\sqrt{2}) + (-3\sqrt{2})(4\sqrt{5}) + (-3\sqrt{2})(3\sqrt{2})$ FOIL

$= 16\sqrt{5^2} + 12\sqrt{10} - 12\sqrt{10} - 9\sqrt{2^2}$ Multiply.

$= 80 - 18$ $\sqrt{5^2} = 5$, $\sqrt{2^2} = 2$

$= 62$ Subtract.

Example 6 Use a Conjugate to Rationalize a Denominator

Simplify $\dfrac{\sqrt{7}}{2 + \sqrt{5}}$.

$\dfrac{\sqrt{7}}{2 + \sqrt{5}} = \dfrac{\sqrt{7}}{(2 + \sqrt{5})} \cdot \dfrac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$ Multiply by $\dfrac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$ because $2 - \sqrt{5}$ is the conjugate of $2 + \sqrt{5}$.

$= \dfrac{2\sqrt{7} - \sqrt{35}}{2^2 - \sqrt{(5)^2}}$ Distributive Property for the numerator
FOIL for the denominator

$= \dfrac{2\sqrt{7} - \sqrt{35}}{4 - 5}$ Multiply.

$= \dfrac{2\sqrt{7} - \sqrt{35}}{-1}$ Simplify.

$= -2\sqrt{7} + \sqrt{35}$ Multiply numerator and denominator by -1.