Lesson 5–7

Example 1 Radical Form

Write each expression in radical form.

a. $36^{\frac{1}{3}}$

$36^{\frac{1}{3}} = \sqrt[3]{36}$ Definition of $b^{\frac{1}{n}}$

b. $z^{\frac{1}{9}}$

$z^{\frac{1}{9}} = \sqrt[9]{z}$ Definition of $b^{\frac{1}{n}}$

Example 2 Exponential Form

Write each radical using rational exponents.

a. $\sqrt[7]{a}$

$\sqrt[7]{a} = a^{\frac{1}{7}}$ Definition of $b^{\frac{1}{n}}$

b. $\sqrt[5]{29}$

$\sqrt[5]{29} = 29^{\frac{1}{5}}$ Definition of $b^{\frac{1}{n}}$

Example 3 Evaluate Expressions with Rational Exponents

Evaluate each expression.

a. $729^{-\frac{1}{6}}$

Method 1

$729^{-\frac{1}{6}} = \frac{1}{729^{\frac{1}{6}}}$ $b^{-n} = \frac{1}{b^n}$

$= \frac{1}{\sqrt[6]{729}}$ $729^{\frac{1}{6}} = \sqrt[6]{729}$

$= \frac{1}{\sqrt[6]{3^6}}$ $729 = 3^6$

$= \frac{1}{3}$ Simplify.

Method 2

$729^{-\frac{1}{6}} = \left(3^6\right)^{-\frac{1}{6}}$ $729 = 3^6$

$= 3^{6\left(-\frac{1}{6}\right)}$ Power of a Power

$= 3^{-1}$ Multiply exponents.

$= \frac{1}{3}$ $3^{-1} = \frac{1}{3^1}$

b. $343^{\frac{2}{3}}$

Method 1

$343^{\frac{2}{3}} = 343^{2\left(\frac{1}{3}\right)}$ Factor.

$= \left(343^2\right)^{\frac{1}{3}}$ Power of a Power

$= \sqrt[3]{343^2}$ $b^{\frac{1}{3}} = \sqrt[3]{b}$

$= \sqrt[3]{\left(7^3\right)^2}$ $343 = 7^3$

$= \sqrt[3]{7^3 \cdot 7^3}$ Expand the square.

$= 7 \cdot 7$ or 49 Find the cube root.

Method 2

$343^{\frac{2}{3}} = \left(7^3\right)^{\frac{2}{3}}$ $343 = 7^3$

$= 7^{3\left(\frac{2}{3}\right)}$ Power of a Power

$= 7^2$ Multiply exponents.

$= 49$ $7^2 = 49$

Example 4 Rational Exponent with Numerator other than 1

SPORTS The volume of the basketball most commonly used in youth basketball leagues is 121.5π cubic inches. The formula for the volume of a sphere, where *r* is the radius, is $V = \frac{4}{3}\pi r^3$ and the formula for the surface area is $A = 4\pi r^2$. Find the surface area of the youth–size basketball. Leave the answer in terms of π.

Explore You are given the volume of the basketball. If you can find the value of r^3, then $r^{\frac{2}{3}}$ can be used in the formula for surface area. (The cube root of r^3 squared will be r^2.)

Plan Substitute the known value for V into the formula $V = \frac{4}{3}\pi r^3$ and solve for r^3. Then substitute $r^{\frac{2}{3}}$ for r^2 into the formula for surface area.

Solve

$V = \frac{4}{3}\pi r^3$	Formula for volume
$121.5\pi = \frac{4}{3}\pi r^3$	Substitute 121.5π for V.
$r^3 = \frac{3}{4}(121.5)$	Divide both sides by π and then multiply by $\frac{3}{4}$.
$r^3 = 91.125$	Simplify.

$A = 4\pi r^2$	Formula for surface area.
$= 4\pi(91.125)^{\frac{2}{3}}$	The cube root of 91.125 is the radius and the formula uses r^2.
$= 4\pi(20.25)$	Use a calculator.
$= 81\pi$	

The surface area of the basketball is 81π in.^{2}

Examine Check by finding the value of r. It is 4.5. 4.5 is close to 5, so $4 \bullet 5^2 = 100\pi$. The answer is reasonable.

Example 5 Simplify Expressions with Rational Exponents

Simplify each expression.

a. $1331^{\frac{1}{12}} \cdot 1331^{\frac{1}{4}}$

$1331^{\frac{1}{12}} \cdot 1331^{\frac{1}{4}} = 1331^{\frac{1}{12}+\frac{1}{4}}$ Multiply powers.

$= 1331^{\frac{1}{3}}$ Add exponents.

$= 11$ $\sqrt[3]{1331} = 11$

b. $m^{-\frac{5}{7}}$

$m^{-\frac{5}{7}} = \dfrac{1}{m^{\frac{5}{7}}}$ $b^{-n} = \dfrac{1}{b^n}$

$= \dfrac{1}{m^{\frac{5}{7}}} \cdot \dfrac{m^{\frac{2}{7}}}{m^{\frac{2}{7}}}$

$= \dfrac{m^{\frac{2}{7}}}{m^{\frac{7}{7}}}$ $m^{\frac{5}{7}} \cdot m^{\frac{2}{7}} = m^{\frac{5}{7}+\frac{2}{7}}$

$= \dfrac{m^{\frac{2}{7}}}{m}$ $m^{\frac{7}{7}} = m^1$ or m

Example 6 Simplify Radical Expressions

Simplify each expression.

a. $\dfrac{\sqrt[3]{1024}}{\sqrt{32}}$

$\dfrac{\sqrt[3]{1024}}{\sqrt{32}} = \dfrac{1024^{\frac{1}{3}}}{32^{\frac{1}{2}}}$ Rational exponents

$= \dfrac{(2^{10})^{\frac{1}{3}}}{(2^5)^{\frac{1}{2}}}$ $1024 = 2^{10}$, $32 = 2^5$

$= \dfrac{2^{\frac{10}{3}}}{2^{\frac{5}{2}}}$ Power of a power

$= 2^{\frac{10}{3}-\frac{5}{2}}$ Quotient of Powers

$= 2^{\frac{5}{6}}$ or $\sqrt[6]{32}$ Simplify.

b. $\sqrt[6]{16a^4}$

$\sqrt[6]{16a^4} = (16a^4)^{\frac{1}{6}}$

$= (2^4 \cdot a^4)^{\frac{1}{6}}$

$= 2^{4\left(\frac{1}{6}\right)} \cdot a^{4\left(\frac{1}{6}\right)}$

$= 2^{\frac{2}{3}} \cdot a^{\frac{2}{3}}$

$= \sqrt[3]{(2a)^2}$ or $\sqrt[3]{4a^2}$

c. $\dfrac{a^{\frac{1}{2}}+4}{a^{\frac{1}{2}}-3}$

$\dfrac{a^{\frac{1}{2}}+4}{a^{\frac{1}{2}}-3} = \dfrac{a^{\frac{1}{2}}+4}{a^{\frac{1}{2}}-3} \cdot \dfrac{a^{\frac{1}{2}}+3}{a^{\frac{1}{2}}+3}$ $a^{\frac{1}{2}} + 3$ is the conjugate of $a^{\frac{1}{2}} - 3$.

$= \dfrac{a + 7a^{\frac{1}{2}} + 12}{a - 9}$ Multiply.