Lesson 5–9

Example 1 Square Roots of Negative Numbers.

Simplify.

a. $\sqrt{-72}$

$$\sqrt{-72} = \sqrt{-1 \cdot 2 \cdot 6^2}$$
$$= \sqrt{-1} \cdot \sqrt{2} \cdot \sqrt{6^2}$$
$$= i \cdot 6 \cdot \sqrt{2} = 6i\sqrt{2}$$

b. $\sqrt{-108b^7}$

$$\sqrt{-108b^7} = \sqrt{-1 \cdot 6^2 \cdot b^6 \cdot 3b}$$
$$= \sqrt{-1} \cdot \sqrt{6^2} \cdot \sqrt{b^6} \cdot \sqrt{3b}$$
$$= i \cdot 6 \cdot b^3 \cdot \sqrt{3b} \text{ or } 6b^3i\sqrt{3b}$$

Example 2 Multiply Pure Imaginary Numbers

Simplify.

a. $(-9i) \cdot (-5i)$

$$(-9i) \cdot (-5i) = 45i^2$$
$$= 45(-1) \quad i^2 = -1$$
$$= -45$$

b. $2\sqrt{-72} \cdot (-3)\sqrt{-50}$

$$2\sqrt{-72} \cdot (-3)\sqrt{-50} = 2i\sqrt{72} \cdot (-3i)\sqrt{50}$$
$$= -6i^2\sqrt{3600}$$
$$= -6i^2\sqrt{60^2}$$
$$= -6(-1)(60)$$
$$= 360$$

Example 3 Simplify a Power of *i*.

Simplify i^{59}.

$i^{59} = i \cdot i^{58}$	Multiplying powers
$= i \cdot (i^2)^{29}$	Power of a power
$= i \cdot (-1)^{29}$	$i^2 = -1$
$= i \cdot (-1)$ or $-i$	$(-1)^{29} = -1$

Example 4 Equation with Imaginary Solutions

Solve $\frac{9}{2}x^2 + 10 = 0$.

$\frac{9}{2}x^2 + 10 = 0$	Original equation
$\frac{9}{2}x^2 = -10$	Subtract 10 from each side.
$x^2 = -\frac{20}{9}$	Multiply each side by $\frac{2}{9}$.
$x = \pm\sqrt{-\frac{20}{9}}$	Take the square root of each side.
$x = \pm\frac{2\sqrt{5}}{3}i$	$\sqrt{-20} = 2\sqrt{5} \cdot i$

Example 5 Equate Complex Numbers

Find the values of a and b that make the equation $a + 4 + (2b - 6)i = 7 + 9i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$a + 4 = 7$	Real parts	$2b - 6 = 9$	Imaginary parts
$a = 3$	Subtract 4 from each side.	$2b = 15$	Add 6 to each side.
		$b = 7.5$	Divide each side by 2.

Example 6 Add and Subtract Complex Numbers

Simplify.

a. $(-7 + 5i) + (12 + 3i)$

$(-7 + 5i) + (12 + 3i)$

$= (-7 + 12) + (5 + 3)i$

$= 5 + 8i$

b. $(6 + 3i) - (-1 - 4i)$

$(6 + 3i) - (-1 - 4i)$

$= [6 - (-1)] + [3 - (-4)]i$

$= 7 + 7i$

Example 7 Multiply Complex Numbers

Simplify.

a. $(1 + 2i)(3 - 4i)$

$(1 + 2i)(3 - 4i)$

$= 1(3) + 1(-4i) + 2i(3) + 2i(-4i)$

$= 3 - 4i + 6i - 8i^2$

$= 3 + 2i - 8(-1)$ $\quad i^2 = -1$

$= 11 + 2i$

b. $(5 + 2i)(5 - 2i)$

$(5 + 2i)(5 - 2i)$

$= 5(5) + 5(-2i) + 2i(5) + 2i(-2i)$

$= 25 - 10i + 10i - 4i^2$

$= 25 - 4(-1)$

$= 29$

Example 8 Divide Complex Numbers

Simplify.

a. $\dfrac{-2}{3+5i}$

$\dfrac{-2}{3+5i} = \dfrac{-2}{3+5i} \cdot \dfrac{3-5i}{3-5i}$ $\quad 3 + 5i$ and $3 - 5i$ are conjugates.

$= \dfrac{-6+10i}{9-25i^2}$ $\quad$ Multiply.

$= \dfrac{-6+10i}{34}$ $\quad i^2 = -1$

$= -\dfrac{3}{17} + \dfrac{5}{17}i$ $\quad$ Standard form

b. $\dfrac{6-7i}{3i}$

$\dfrac{6-7i}{3i} = \dfrac{6-7i}{3i} \cdot \dfrac{i}{i}$ $\quad$ Multiply by $\dfrac{i}{i}$.

$= \dfrac{6i-7i^2}{3i^2}$ $\quad$ Multiply.

$= \dfrac{6i+7}{-3}$ $\quad i^2 = -1$

$= -\dfrac{7}{3} - 2i$ $\quad$ Standard form