

Lesson 6-3

Example 1 Two Roots

Solve each equation by factoring.

a. $5x^2 = -2x$

$$5x^2 = -2x$$

$$5x^2 + 2x = 0$$

$$x(5x + 2) = 0$$

$$x = 0 \quad \text{or} \quad 5x + 2 = 0$$

$$5x = -2$$

$$x = -\frac{2}{5}$$

Original equation

Add $2x$ to each side.

Factor the binomial.

Zero Product Property

Solve the second equation.

The solution set is $\left\{-\frac{2}{5}, 0\right\}$.

CHECK: Substitute $-\frac{2}{5}$ and 0 for x in the original equation.

$$5\left(-\frac{2}{5}\right)^2 \stackrel{?}{=} -2\left(-\frac{2}{5}\right)$$
$$\frac{4}{5} = \frac{4}{5} \quad \checkmark$$

$$5x^2 = -2x$$
$$5(0)^2 \stackrel{?}{=} -2(0)$$
$$0 = 0 \quad \checkmark$$

b. $3x^2 - 2 = 5x$

$$3x^2 - 2 = 5x$$

$$3x^2 - 5x - 2 = 0$$

$$(3x + 1)(x - 2) = 0$$

$$3x + 1 = 0 \quad \text{or} \quad x - 2 = 0$$

$$3x = -1 \quad \quad \quad x = 2$$

$$x = -\frac{1}{3}$$

Original equation

Subtract $5x$ from each side.

Factor the trinomial.

Zero Product Property

Solve each equation.

The solution set is $\left\{-\frac{1}{3}, 2\right\}$. Check each solution.

Example 2 Double Root

Solve $x^2 + 10x + 25 = 0$ by factoring.

$$x^2 + 10x + 25 = 0$$

$$(x + 5)(x + 5) = 0$$

$$x + 5 = 0 \quad \text{or} \quad x + 5 = 0$$

$$x = -5 \quad \quad \quad x = -5$$

Original equation

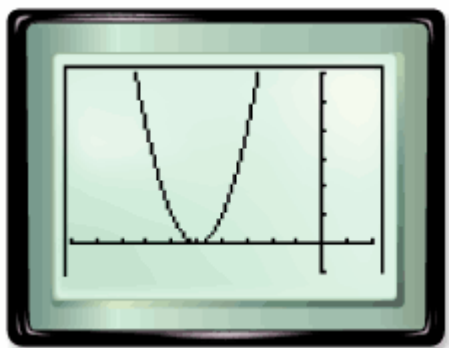
Factor.

Zero Product Property

Solve each equation.

The solution is $\{-5\}$.

CHECK: The graph of the related function, $f(x) = x^2 + 10x + 25$, intersects the x -axis only once. Since the zero of the function is -5 , the solution of the related equation is -5 .



Example 3 Greatest Common Factor

Multiple Choice Test Item

What is the negative solution of the equation $-8x^2 - 4x + 12 = 0$?

- A. $-\frac{3}{2}$ B. -3 C. $-\frac{2}{3}$ D. 1 E. 3

Read the Test Item

You are asked to find the *negative* solution of the given quadratic equation. This implies that the equation also has a solution that is not negative. Since a quadratic equation can either have one, two, or no solutions, you should expect to find two solutions to this equation.

Solve the Test Item

Solve this equation by factoring. Before trying to factor $-8x^2 - 4x + 12$ into two binomials, look for a greatest common factor. Notice that each term is divisible by -4 .

$-8x^2 - 4x + 12 = 0$	Original equation
$-4(2x^2 + x - 3) = 0$	Factor.
$2x^2 + x - 3 = 0$	Divide each side by -4 .
$(2x + 3)(x - 1) = 0$	Factor.
$2x + 3 = 0$ or $x - 1 = 0$	Zero Product Property
$2x = -3$ $x = 1$	Solve each equation.
$x = -\frac{3}{2}$	

Both solutions, $-\frac{3}{2}$ and 1 , are listed among the answer choices. Since the question asked for the negative solution, the answer is A.

Example 4 Write an Equation Given Roots

Write a quadratic equation with $-\frac{3}{4}$ and 1 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a , b , and c are integers.

$(x - p)(x - q) = 0$	Write the pattern.
$\left[x - \left(-\frac{3}{4} \right) \right] (x - 1) = 0$	Replace p with $-\frac{3}{4}$ and q with 1.
$\left(x + \frac{3}{4} \right) (x - 1) = 0$	Simplify.
$x^2 - \frac{1}{4}x - \frac{3}{4} = 0$	Use FOIL.
$4x^2 - x - 3 = 0$	Multiply each side by 4 so that b and c are integers.

A quadratic equation with roots $-\frac{3}{4}$ and 1 and integral coefficients is $4x^2 - x - 3 = 0$. You can check this result by graphing the related function, $f(x) = 4x^2 - x - 3$.