

### Lesson 8-3

#### Example 1 Write an Equation Given the Center and Radius

Write an equation for the circle with center at (5, -3) and radius 11.

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 && \text{Equation of a circle} \\(x-5)^2 + [y-(-3)]^2 &= 11^2 && (h, k) = (5, -3), r = 11 \\(x-5)^2 + (y+3)^2 &= 121 && \text{Simplify.}\end{aligned}$$

The equation is  $(x-5)^2 + (y+3)^2 = 121$ .

#### Example 2 Write an Equation Given a Diameter

Write an equation for a circle if the endpoints of a diameter are at (-3, -7) and (2, 2).

*Explore* To write an equation of a circle, you must know the center and the radius.

*Plan* You can find the center of the circle by finding the midpoint of the diameter. Then you can find the radius of the circle by finding the distance from the center to one of the given points.

$$\begin{aligned}\text{Solve } (h, k) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) && \text{Midpoint Formula} \\ &= \left( \frac{-3 + 2}{2}, \frac{-7 + 2}{2} \right) && (x_1, y_1) = (-3, -7), (x_2, y_2) = (2, 2) \\ &= \left( -\frac{1}{2}, -\frac{5}{2} \right) && \text{Simplify.}\end{aligned}$$
$$\begin{aligned}r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{\left[ -\frac{1}{2} - (-3) \right]^2 + \left[ -\frac{5}{2} - (-7) \right]^2} && (x_1, y_1) = (-3, -7), (x_2, y_2) = \left( -\frac{1}{2}, -\frac{5}{2} \right) \\ &= \sqrt{\left( \frac{5}{2} \right)^2 + \left( \frac{9}{2} \right)^2} && \text{Subtract.} \\ &= \sqrt{\frac{106}{4}} && \text{Simplify.}\end{aligned}$$

The radius of the circle is  $\sqrt{\frac{106}{4}}$  units, so  $r^2 = \frac{106}{4}$  or  $\frac{53}{2}$ . An equation of the circle is  $\left( x + \frac{1}{2} \right)^2$

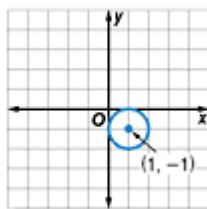
$$+ \left( y + \frac{5}{2} \right)^2 = \frac{53}{2}.$$

*Examine* Each of the given points satisfies the equation, so the equation is reasonable.

### Example 3 Write an Equation Given the Center and a Tangent

Write an equation for a circle with center at  $(1, -1)$  that is tangent to the  $y$ -axis.

Sketch the circle. Since the circle is tangent to the  $y$ -axis, its radius is 1.

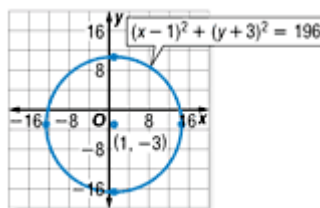


An equation of the circle is  $(x - 1)^2 + (y + 1)^2 = 1$ .

### Example 4 Graph an Equation in Standard Form

Find the center and radius of the circle with equation  $(x - 1)^2 + (y + 3)^2 = 196$ . Then graph the circle.

Rewrite the equation as  $(x - 1)^2 + [y - (-3)]^2 = 14^2$ . The center of the circle is  $(1, -3)$  and the radius is 14.



The table lists some integer values for  $x$  and  $y$  that satisfy the equation.

$x$	$y$
1	11
1	-17
15	-3
-13	-3

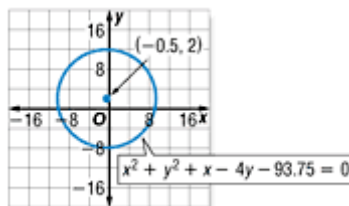
Graph all of these points and draw the circle that passes through them.

### Example 5 Graph an Equation not in Standard Form

Find the center and radius of the circle with equation  $x^2 + y^2 + x - 4y - 93.75 = 0$ . Then graph the circle.

Complete the square.

$$\begin{aligned}x^2 + y^2 + x - 4y - 93.75 &= 0 \\x^2 + x + \quad + y^2 - 4y + \quad &= 93.75 + \quad + \quad \\x^2 + x + 0.25 + y^2 - 4y + 4 &= 93.75 + 0.25 + 4 \\(x + 0.5)^2 + (y - 2)^2 &= 98\end{aligned}$$



The center of the circle is at  $(-0.5, 2)$ , and the radius is  $7\sqrt{2}$ . Locate the center and then find several points located about 9.9 units from the center. Draw the circle that passes through them.