

Lesson 9-1

Example 1 Simplify a Rational Expression

a. Simplify $\frac{2x^2 + x - 6}{5x + 10}$.

Since you cannot identify any common factors in the numerator and denominator, factor each of the expressions.

$$\frac{2x^2 + x - 6}{5x + 10} = \frac{(2x - 3)(x + 2)}{5(x + 2)}$$

Factor the numerator and denominator.

$$= \frac{(2x - 3) \overset{1}{\cancel{(x + 2)}}}{5 \underset{1}{\cancel{(x + 2)}}}$$

Divide the numerator and denominator by the common

factor of $(x + 2)$.

$$= \frac{2x - 3}{5}$$

Simplify.

b. Under what conditions is this expression undefined?

Just as with a fraction, a rational expression is undefined if the denominator is equal to 0. Consider the original denominator, $5x + 10$. The value that would make the denominator equal to 0 is -2 . So, the expression is undefined when $x = -2$.

Example 2 Use the Process of Elimination

Multiple-Choice Test Item

For what value(s) of a is $\frac{a^2 + 2a - 15}{a^2 - 3a}$ undefined?

A. $-5, 3$

B. $-3, 5$

C. $-5, 0, 3$

D. 0

E. $0, 3$

Read the Test Item

You want to determine which values of a make the denominator equal to 0.

Solve the Test Item

Look at the possible answers. Notice that a is in both terms of the expression in the denominator. That means that $a = 0$ will make the denominator equal to 0. Therefore, you can eliminate choices A and B since they do not contain an answer of 0. Since the denominator is a quadratic expression, it can have only 2 possible values. That eliminates choice C. Determine which choice D or E is correct.

Since $3^2 - 3(3) = 0$, the answer is E.

Example 3 Simplify by Factoring Out –1

Simplify $\frac{4 - b^2}{b^2 + 3b - 10}$.

$$\frac{4 - b^2}{b^2 + 3b - 10} = \frac{(2 - b)(2 + b)}{(b + 5)(b - 2)}$$

Factor the numerator and the denominator.

$$= \frac{(-1) \overset{1}{\cancel{(b-2)}}(b+2)}{(b+5) \overset{1}{\cancel{(b-2)}}$$

$$2 - b = -(-2 + b) \text{ or } -1(b - 2)$$

$$= \frac{(-1)(b + 2)}{(b + 5)} \text{ or } \frac{-b - 2}{b + 5}$$

Simplify.

Example 4 Multiply Rational Expressions

Simplify $\frac{2xy}{5x^2z} \cdot \frac{15x}{4z}$.

$$\frac{2xy}{5x^2z} \cdot \frac{15x}{4z} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot y \cdot 3 \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot z \cdot \underset{1}{\cancel{x}} \cdot 2 \cdot z}$$

Factor.

$$= \frac{3 \cdot y}{2 \cdot z \cdot z}$$

Simplify.

$$= \frac{3y}{2z^2}$$

Simplify.

Example 5 Divide Rational Expressions

Simplify $\frac{6a^3c^2}{5b^2} \div \frac{3c^2}{25a}$.

$$\frac{6a^3c^2}{5b^2} \div \frac{3c^2}{25a} = \frac{6a^3c^2}{5b^2} \cdot \frac{25a}{3c^2}$$

Multiply by the reciprocal of the divisor.

$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot a \cdot a \cdot a \cdot \overset{1}{\cancel{c}} \cdot \overset{1}{\cancel{c}} \cdot \overset{1}{\cancel{3}} \cdot 5 \cdot a}{\underset{1}{\cancel{3}} \cdot b \cdot b \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{c}} \cdot \underset{1}{\cancel{c}}}$$

Factor.

$$= \frac{2 \cdot 5 \cdot a \cdot a \cdot a \cdot a}{b \cdot b}$$

Simplify.

$$= \frac{10a^4}{b^2}$$

Simplify.

Example 6 Polynomials in the Numerator and Denominator

Simplify $\frac{3x+6}{2x-10} \div \frac{4x^2+7x-2}{2x^2-7x-15}$.

$$\begin{aligned} \frac{3x+6}{2x-10} \div \frac{4x^2+7x-2}{2x^2-7x-15} &= \frac{3x+6}{2x-10} \cdot \frac{2x^2-7x-15}{4x^2+7x-2} && \text{Multiply by the reciprocal of the divisor.} \\ &= \frac{\overset{1}{3} \cancel{(x+2)}}{\overset{1}{2} \cancel{(x-5)}} \cdot \frac{\overset{1}{(2x+3)} \cancel{(x-5)}}{\cancel{(x+2)} \overset{1}{(4x-1)}} && \text{Factor.} \\ &= \frac{3(2x+3)}{2(4x-1)} && \text{Simplify.} \\ &= \frac{6x+9}{8x-2} \end{aligned}$$

Example 7 Simplify a Complex Fraction

Simplify $\frac{\frac{z-1}{z^2-1}}{\frac{z^2-2z+1}{z^2-z-2}}$.

$$\begin{aligned} \frac{\frac{z-1}{z^2-1}}{\frac{z^2-2z+1}{z^2-z-2}} &= \frac{z-1}{z^2-1} \div \frac{z^2-2z+1}{z^2-z-2} && \text{Express as a division expression.} \\ &= \frac{z-1}{z^2-1} \cdot \frac{z^2-z-2}{z^2-2z+1} && \text{Multiply by the reciprocal of the divisor.} \\ &= \frac{\overset{1}{\cancel{(z-1)}}}{\cancel{(z-1)} \overset{1}{\cancel{(z+1)}}} \cdot \frac{\overset{1}{(z-2)} \cancel{(z+1)}}{\cancel{(z-1)} \overset{1}{(z-1)}} && \text{Factor.} \\ &= \frac{\overset{1}{z-2}}{\overset{1}{(z-1)^2}} && \text{Simplify.} \end{aligned}$$