Lesson 9–2

Example 1 LCM of Monomials

Find the LCM of $10ab^2$, $105a^3b^2c$, and $50ab^2c^3$.

$10ab^2 = 2 \cdot 5 \cdot a \cdot b^2$	Factor the first monomial.
$105a^3b^2c = 3 \cdot 5 \cdot 7 \cdot a^3 \cdot b^2 \cdot c$	Factor the second monomial.
$50ab^2c^3 = 2 \cdot 5^2 \cdot a \cdot b^2 \cdot c^3$	Factor the third monomial.
$\text{LCM} = 2 \cdot 3 \cdot 5^2 \cdot 7 \cdot a^3 \cdot b^2 \cdot c^3$	Use each factor the greatest number of times
$= 1050a^3b^2c^3$	it appears as a factor and simplify.

Example 2 LCM of Polynomials

Find the LCM of $2x^3 - 9x^2 - 5x$ and $x^4 - 10x^3 + 25x^2$.

$2x^3 - 9x^2 - 5x = x(2x + 1)(x - 5)$	Factor the first polynomial.
$x^4 - 10x^3 + 25x^2 = x^2(x - 5)^2$	Factor the second polynomial.
$\text{LCM} = x^2(2x + 1)(x - 5)^2$	Use each factor the greatest number of times it appears as a factor.

Example 3 Monomial Denominators

Simplify $\dfrac{3a}{10b^2c} - \dfrac{4b^2}{25a^2c^2}$.

$$\frac{3a}{10b^2c} - \frac{4b^2}{25a^2c^2} = \frac{3a \cdot 5a^2c}{10b^2c \cdot 5a^2c} - \frac{4b^2 \cdot 2b^2}{25a^2c^2 \cdot 2b^2}$$

The LCD is $50a^2b^2c^2$. Find equivalent fractions that have this denominator.

$$= \frac{15a^3c}{50a^2b^2c^2} - \frac{8b^4}{50a^2b^2c^2}$$

Simplify each numerator and denominator.

$$= \frac{15a^3c - 8b^4}{50a^2b^2c^2}$$

Subtract the numerators.

Example 4 Polynomial Denominators

Simplify $\dfrac{-4}{2a^2-2a-4}+\dfrac{6}{a^2+4a+3}$.

$$\frac{-4}{2a^2-2a-4}+\frac{6}{a^2+4a+3}$$

$$=\frac{-4}{2(a-2)(a+1)}+\frac{6}{(a+3)(a+1)}$$ Factor the denominators.

$$=\frac{-4(a+3)}{2(a-2)(a+1)(a+3)}+\frac{6(2)(a-2)}{2(a-2)(a+1)(a+3)}$$ The LCD is $2(a-2)(a+1)(a+3)$.

$$=\frac{-4a-12}{2(a-2)(a+1)(a+3)}+\frac{12a-24}{2(a-2)(a+1)(a+3)}$$ Distributive Property

$$=\frac{-4a-12+12a-24}{2(a-2)(a+1)(a+3)}$$ Add the numerators.

$$=\frac{8a-36}{2(a-2)(a+1)(a+3)}$$ Combine like terms.

$$=\frac{4(2a-9)}{2(a-2)(a+1)(a+3)} \text{ or } \frac{2(2a-9)}{(a-2)(a+1)(a+3)}$$ Simplify.

Example 5 Simplify Complex Fractions

Simplify $\dfrac{\dfrac{1}{a-2}+\dfrac{2}{a+1}}{\dfrac{3}{a^2-a-2}}$.

$$\frac{\frac{1}{a-2}+\frac{2}{a+1}}{\frac{3}{a^2-a-2}}=\frac{\frac{1(a+1)}{(a-2)(a+1)}+\frac{2(a-2)}{(a-2)(a+1)}}{\frac{3}{a^2-a-2}}$$ The LCD of the numerator is $(a-2)(a+1)$.

$$=\frac{\frac{3a-3}{(a-2)(a+1)}}{\frac{3}{a^2-a-2}}$$ Simplify the numerator.

$$=\frac{3a-3}{(a-2)(a+1)}\div\frac{3}{a^2-a-2}$$ Write as a division expression.

$$=\frac{3a-3}{(a-2)(a+1)}\cdot\frac{a^2-a-2}{3}$$ Multiply by the reciprocal of the divisor.

$$=\frac{\overset{1}{\cancel{3}}(a-1)}{\underset{1}{\cancel{(a-2)}}\underset{1}{\cancel{(a+1)}}}\cdot\frac{\overset{1}{\cancel{(a-2)}}\overset{1}{\cancel{(a+1)}}}{\underset{1}{\cancel{3}}}$$ Simplify.

$$=a-1$$

Example 6 Use a Complex Fraction to Solve a Problem

GEOMETRY An expression for the area of a parallelogram in terms of a and b is $\frac{a^2 + 2ab + b^2}{2}$. An expression for the length of the base is $\frac{a+b}{5}$. Find the height of the parallelogram.

$A = bh$ Formula for the area of a parallelogram

$h = \frac{A}{b}$ Solve the formula for h.

$= \frac{\frac{a^2 + 2ab + b^2}{2}}{\frac{a + b}{5}}$ $A = \frac{a^2 + 2ab + b^2}{2}$, $b = \frac{a + b}{5}$

$= \frac{a^2 + 2ab + b^2}{2} \div \frac{a + b}{5}$ Write as a division expression.

$= \frac{a^2 + 2ab + b^2}{2} \cdot \frac{5}{a + b}$ Multiply by the reciprocal of the divisor.

$= \frac{(a+b)(a+b)}{2} \cdot \frac{5}{a+b}$ Factor.

$= \frac{5(a + b)}{2}$ Simplify.

The height of the parallelogram is $\frac{5(a + b)}{2}$.