Lesson 9–3

Example 1 Vertical Asymptotes and Point Discontinuity

Determine the equations of any vertical asymptotes and the values of *x* for any holes in the graph of $f(x) = \frac{x^2 - x - 6}{x^2 + 3x - 18}$.

First, factor the numerator and denominator of the rational expression.

$$\frac{x^2 - x - 6}{x^2 + 3x - 18} = \frac{(x - 3)(x + 2)}{(x - 3)(x + 6)}$$

The function is undefined for $x = -6$ and $x = 3$.

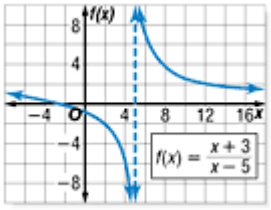
Since $\frac{\cancel{(x-3)}^{1}(x+2)}{\cancel{(x-3)}_{1}(x+6)} = \frac{x + 2}{x + 6}$, $x = -6$ is a vertical asymptote, and $x = 3$ represents a hole in the graph.

Example 2 Graph with a Vertical Asymptote

Graph $f(x) = \frac{x + 3}{x - 5}$.

The function is undefined for $x = 5$. Since $\frac{x + 3}{x - 5}$ is in simplest form, $x = 5$ is a vertical asymptote. Draw the vertical asymptote. Make a table of values. Plot the points and draw the graph.

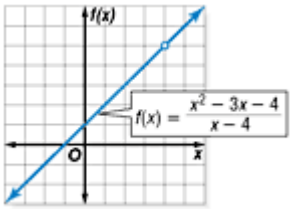
x	*f*(*x*)
–5	0.2
–3	0
–1	–0.3
0	–0.6
1	–1
3	–3
7	5
9	3
11	2.3
13	2
15	1.8



As the *x* values for the left portion of the graph decrease, it appears that the *y* values get closer and closer to 1. As the *x* values for the right portion of the graph increase, it appears that the *y* values get closer and closer to 1. The line with equation $f(x) = 1$ is a horizontal asymptote of the function.

Example 3 Graph with Point Discontinuity

Graph $f(x) = \dfrac{x^2 - 3x - 4}{x - 4}$.



Notice that $\dfrac{x^2 - 3x - 4}{x - 4} = \dfrac{(x - 4)(x + 1)}{x - 4}$ or $x + 1$. Therefore, the graph of $f(x) = \dfrac{x^2 - 3x - 4}{x - 4}$ is the graph of $f(x) = x + 1$ with a hole at $x = 4$.

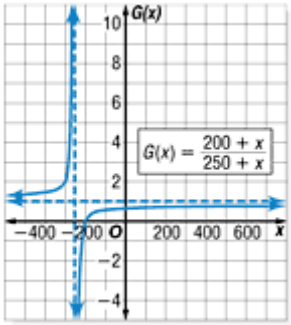
Example 4 Use Graphs of Rational Functions

GRADES Tyler has earned 200 of 250 points possible so far in his World History class. He wants to increase his current average of 80% in this class. If he can earn every point possible by the end of the grading period, his average can be determined using $G(x) = \dfrac{200 + x}{250 + x}$.

a. Graph the function.

The function is undefined for $x = -250$. Since $\dfrac{200 + x}{250 + x}$ is in simplest form, $x = -250$ is a vertical asymptote. Draw the vertical asymptote. Make a table of values. Plot the points and draw the graph.

x	$f(x)$
–300	2
–260	6
–255	11
–240	–4
–200	0
–100	0.7
0	0.8
100	0.86
300	0.91
500	0.93
700	0.95



b. What is the $G(x)$ intercept of the graph? What does it represent?

The $G(x)$ intercept is 0.8. It represents Tyler's current average of 0.80 or 80%.

c. What part of the graph is meaningful in the context of the problem?

The part of the graph in the first quadrant is meaningful in the context of the problem, because Tyler cannot earn negative points in his class.

d. What is the equation of the horizontal asymptote? Explain its meaning in relationship to this problem.

The equation of the horizontal asymptote is $G(x) = 1$. The asymptote shows that Tyler's grade cannot exceed 100%, since 100% = 1.