

## Lesson 9-4

### Example 1 Direct Variation

If  $y$  varies directly as  $x$  and  $y = -240$  when  $x = 75$ , find  $y$  when  $x = 32$ .

$$\begin{aligned}\frac{y_1}{x_1} &= \frac{y_2}{x_2} && \text{Direct proportion} \\ \frac{-240}{75} &= \frac{y_2}{32} && y_1 = -240, x_1 = 75, \text{ and } x_2 = 32 \\ -240(32) &= 75(y_2) && \text{Cross multiply.} \\ -7680 &= 75y_2 && \text{Simplify.} \\ -102.4 &= y_2 && \text{Divide each side by 75.}\end{aligned}$$

When  $x = 32$ , the value of  $y$  is  $-102.4$ .

### Example 2 Joint Variation

Suppose  $y$  varies jointly as  $x$  and  $z$ . Find  $y$  when  $x = 6$  and  $z = 30$ , if  $y = 7$  when  $z = 10$  and  $x = 3$ .

Use a proportion that relates the values.

$$\begin{aligned}\frac{y_1}{x_1 z_1} &= \frac{y_2}{x_2 z_2} && \text{Joint variation} \\ \frac{7}{3(10)} &= \frac{y_2}{6(30)} && y_1 = 7, x_1 = 3, z_1 = 10, x_2 = 6, \text{ and } z_2 = 30 \\ 7(6)(30) &= 3(10)(y_2) && \text{Cross multiply.} \\ 1260 &= 30y_2 && \text{Simplify.} \\ 42 &= y_2 && \text{Divide each side by 30.}\end{aligned}$$

When  $x = 6$  and  $z = 30$ , the value of  $y$  is  $42$ .

### Example 3 Inverse Variation

If  $a$  varies inversely as  $b$  and  $a = -102$  when  $b = 68$ , find  $a$  when  $b = 3$ .

$$\begin{aligned}\frac{a_1}{b_2} &= \frac{a_2}{b_1} && \text{Inverse variation} \\ \frac{-102}{3} &= \frac{a_2}{68} && a_1 = -102, b_1 = 68, \text{ and } b_2 = 3 \\ -102(68) &= 3(a_2) && \text{Cross multiply.} \\ -6936 &= 3a_2 && \text{Simplify.} \\ -2312 &= a_2 && \text{Divide each side by 3.}\end{aligned}$$

When  $b = 3$ , the value of  $a$  is  $-2312$ .

**Example 4 Use Inverse Variation**

**GEOMETRY** The volume of any cylinder can be found by using the formula  $V = \pi r^2 h$ , where  $r$  is the radius of the cylinder and  $h$  is its height.

a. Solve the formula for  $h$ .

$$V = \pi r^2 h \quad \text{Original formula}$$

$$h = \frac{V}{\pi r^2} \quad \text{Divide each side by } \pi r^2.$$

b. How does  $h$  vary with respect to  $r^2$ ?

$h$  varies inversely as  $r^2$  since there is some nonzero constant ( $V \div \pi$ ) such that  $h = \frac{(V \div \pi)}{r^2}$ .

c. If the radius of a cylinder is 2 in. and the volume is  $6\pi$  in. , what is the height?

$$h = \frac{V}{\pi r^2} \quad \text{Formula from part a}$$

$$= \frac{6\pi}{\pi(2)^2} \quad r = 2, V = 6\pi$$

$$= 1.5 \quad \text{Simplify.}$$

The height of the cylinder is 1.5 in.