

Factoring Using the Distributive Property

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The process of finding the factors of a polynomial when you know the product is called **factoring**. You can use the Distributive Property to factor a polynomial. If you know a product and one of its factors, you can use division to find the other factor. When the terms of a binomial have no common factors other than 1, it is a prime polynomial.

EXAMPLES

A Factor $9a^2b^3 - 3ab^2 + 6ab$.

$$\begin{aligned} 9a^2b^3 &= \left(\begin{array}{c} 3 \\ 3 \\ 2 \end{array} \right) \cdot 3 \cdot \left(\begin{array}{c} a \\ a \\ a \end{array} \right) \cdot a \cdot \left(\begin{array}{c} b \\ b \\ b \end{array} \right) \cdot b \cdot b \cdot b \\ 3ab^2 &= \left(\begin{array}{c} 3 \\ 3 \\ 2 \end{array} \right) \cdot 3 \cdot \left(\begin{array}{c} a \\ a \\ a \end{array} \right) \cdot a \cdot \left(\begin{array}{c} b \\ b \\ b \end{array} \right) \cdot b \cdot b \\ 6ab &= 2 \cdot \left(\begin{array}{c} 3 \\ 3 \\ 2 \end{array} \right) \cdot 3 \cdot \left(\begin{array}{c} a \\ a \\ a \end{array} \right) \cdot a \cdot \left(\begin{array}{c} b \\ b \\ b \end{array} \right) \cdot b \end{aligned}$$

The GCF is $3ab$.

Use the Distributive Property to express the polynomial as the product of the GCF and the remaining factor of each term.

$$\begin{aligned} 9a^2b^3 - 3ab^2 + 6ab &= 3ab(3ab^2) - 3ab(b) + 3ab(2) \\ &= 3ab(3ab^2 - b + 2) \end{aligned}$$

You can check this answer by using the Distributive Property. $3ab(3ab^2 - b + 2) = 9a^2b^3 - 3ab^2 + 6ab$

B Divide $8x^2 - 6x$ by $2x$.

Divide each term by $2x$.

$$\begin{aligned} (8x^2 - 6x) \div 2x &= \frac{8x^2}{2x} - \frac{6x}{2x} \\ &= \frac{\overset{4}{\cancel{8}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{x}}} - \frac{\overset{3}{\cancel{6}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{x}}} \\ &= 4x - 3 \end{aligned}$$

So, $(8x^2 - 6x) \div 2x = 4x - 3$.

PRACTICE

Factor each polynomial. If the polynomial cannot be factored, write prime.

- | | | |
|-----------------------------|----------------------|--------------------------------|
| 1. $7b^2 + 42b$ | 2. $15m^2n - 27mn^2$ | 3. $10xz^2 + 3y$ |
| 4. $8s^3 + 24s^2q$ | 5. $16g + 14gh^2$ | 6. $36k^5 + 24jk^3$ |
| 7. $10a^3b - 5abc + 15ab^2$ | 8. $3x^3 + 2y^2 + z$ | 9. $28cd^2 + 7c^2d^3 - 14c^3d$ |

Find each quotient.

- | | |
|-----------------------------------|------------------------------------|
| 10. $(12x^2 + 9x) \div 3x$ | 11. $(28a^2b - 21a) \div 7$ |
| 12. $(16mn^2 - 20m^2n) \div 4mn$ | 13. $(5xyz + y) \div y$ |
| 14. $(15a^2b^2 + 5ab^3) \div 5ab$ | 15. $(12g^3h - 18g^2h^2) \div 6gh$ |

16. Geometry The area of a rectangle is $(15x^2 + 4x)$ square feet. If the length is x feet, find the width.

17. Standardized Test Practice Factor the polynomial $4wf + 8w$.

- A** $4(wf + 2)$ **B** $4w(f + 2)$ **C** $4w(f + 8)$ **D** $w(4f + 8)$

Answers: 1. $7b(b + 6)$ 2. $3mn(5m - 9n)$ 3. prime 4. $2g(8 + 3q)$ 5. $2g(8 + 7h^2)$ 6. $12k^3(3k^2 + 2l)$ 7. $5ab(2a^2 - c + 3b)$ 8. prime 9. $7cd(4d + cd^2 - 2c^2)$ 10. $4x + 3$ 11. $4a^2b - 3a$ 12. $4n - 5m$ 13. $5xz + 1$ 14. $3ab + b^2$ 15. $2g^2 - 3gh$ 16. $(15x + 4)$ ft 17. B