

Factors (Pages 420–425)

Recall that when two or more numbers are multiplied, each number is a *factor* of the product. *Prime numbers* are whole numbers greater than 1 that have exactly two factors, the number itself and 1. Whole numbers that have more than two factors are called *composite numbers*. When a number is expressed as a product of prime factors, the expression is called the *prime factorization* of the number. A monomial is in *factored form* when it is expressed as the product of prime numbers and variables and no variable has an exponent greater than 1. You can use prime factorization to find the **greatest common factor (GCF)** of two or more integers, which is the product of the prime factors common to the integers.

EXAMPLES

A Factor $-21xy^2$.

To factor a negative integer, first express it as a product of a whole number and -1 . Then find the prime factorization.

$$\begin{aligned} -21xy^2 &= -1 \cdot 21xy^2 \\ &= -1 \cdot 3 \cdot 7 \cdot x \cdot y \cdot y \end{aligned}$$

B Find the GCF of $24a^2$ and $36a$.

Write each monomial in factored form, then circle the common factors.

$$\begin{aligned} 24a^2 &= \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot \textcircled{3} \cdot a \cdot a \\ 36a &= \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{3} \cdot 3 \cdot a \end{aligned}$$

The GCF is the product of the common factors.

$$2 \cdot 2 \cdot 3 \cdot a = 12a$$

The GCF is $12a$.

PRACTICE

Find the factors of each number. Then classify each number as prime or composite.

1. 15 2. 27 3. 23 4. 31 5. 42 6. 67

Factor each monomial.

7. $30m^3$ 8. $-63k^2p$ 9. $28x^2y^2$ 10. $72rs$

Find the GCF of each set of numbers or monomials.

11. 18, 50 12. 12, 28 13. 56, 126
14. $5a^3, 20a$ 15. $12x^2y^4, 18y^2z$ 16. $4c, 16c^2, 28b^2c^5$

17. The Classroom Ms. Yip has 32, 36, and 24 students in each of her morning classes. What is the greatest number of desks she can place in each row of desks so that no row will be partially filled when the students from each of her classes are seated?



18. Standardized Test Practice Which list shows all the factors of 98?

- A** 1, 2, 4, 7, 14, 49 **B** 1, 2, 7, 8, 14, 49 **C** 1, 2, 7, 18, 49, 98 **D** 1, 2, 7, 14, 49, 98

Answers: 1. 1, 3, 5, 15; composite 2. 1, 3, 9, 27; composite 3. 1, 23; prime 4. 1, 31; prime 5. 1, 2, 3, 6, 7, 14, 21, 42; composite
6. 1, 67; prime 7. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 65, 69, 73, 77, 83, 87, 91, 95, 97, 101, 103, 107, 109, 113, 117, 121, 125, 129, 133, 137, 141, 145, 149, 151, 155, 159, 163, 167, 171, 175, 179, 183, 187, 191, 195, 199, 203, 207, 211, 215, 219, 223, 227, 231, 235, 239, 243, 247, 251, 255, 259, 263, 267, 271, 275, 279, 283, 287, 291, 295, 299, 303, 307, 311, 315, 319, 323, 327, 331, 335, 339, 343, 347, 351, 355, 359, 363, 367, 371, 375, 379, 383, 387, 391, 395, 399, 403, 407, 411, 415, 419, 423, 427, 431, 435, 439, 443, 447, 451, 455, 459, 463, 467, 471, 475, 479, 483, 487, 491, 495, 499, 503, 507, 511, 515, 519, 523, 527, 531, 535, 539, 543, 547, 551, 555, 559, 563, 567, 571, 575, 579, 583, 587, 591, 595, 599, 603, 607, 611, 615, 619, 623, 627, 631, 635, 639, 643, 647, 651, 655, 659, 663, 667, 671, 675, 679, 683, 687, 691, 695, 699, 703, 707, 711, 715, 719, 723, 727, 731, 735, 739, 743, 747, 751, 755, 759, 763, 767, 771, 775, 779, 783, 787, 791, 795, 799, 803, 807, 811, 815, 819, 823, 827, 831, 835, 839, 843, 847, 851, 855, 859, 863, 867, 871, 875, 879, 883, 887, 891, 895, 899, 903, 907, 911, 915, 919, 923, 927, 931, 935, 939, 943, 947, 951, 955, 959, 963, 967, 971, 975, 979, 983, 987, 991, 995, 999, 1003, 1007, 1011, 1015, 1019, 1023, 1027, 1031, 1035, 1039, 1043, 1047, 1051, 1055, 1059, 1063, 1067, 1071, 1075, 1079, 1083, 1087, 1091, 1095, 1099, 1103, 1107, 1111, 1115, 1119, 1123, 1127, 1131, 1135, 1139, 1143, 1147, 1151, 1155, 1159, 1163, 1167, 1171, 1175, 1179, 1183, 1187, 1191, 1195, 1199, 1203, 1207, 1211, 1215, 1219, 1223, 1227, 1231, 1235, 1239, 1243, 1247, 1251, 1255, 1259, 1263, 1267, 1271, 1275, 1279, 1283, 1287, 1291, 1295, 1299, 1303, 1307, 1311, 1315, 1319, 1323, 1327, 1331, 1335, 1339, 1343, 1347, 1351, 1355, 1359, 1363, 1367, 1371, 1375, 1379, 1383, 1387, 1391, 1395, 1399, 1403, 1407, 1411, 1415, 1419, 1423, 1427, 1431, 1435, 1439, 1443, 1447, 1451, 1455, 1459, 1463, 1467, 1471, 1475, 1479, 1483, 1487, 1491, 1495, 1499, 1503, 1507, 1511, 1515, 1519, 1523, 1527, 1531, 1535, 1539, 1543, 1547, 1551, 1555, 1559, 1563, 1567, 1571, 1575, 1579, 1583, 1587, 1591, 1595, 1599, 1603, 1607, 1611, 1615, 1619, 1623, 1627, 1631, 1635, 1639, 1643, 1647, 1651, 1655, 1659, 1663, 1667, 1671, 1675, 1679, 1683, 1687, 1691, 1695, 1699, 1703, 1707, 1711, 1715, 1719, 1723, 1727, 1731, 1735, 1739, 1743, 1747, 1751, 1755, 1759, 1763, 1767, 1771, 1775, 1779, 1783, 1787, 1791, 1795, 1799, 1803, 1807, 1811, 1815, 1819, 1823, 1827, 1831, 1835, 1839, 1843, 1847, 1851, 1855, 1859, 1863, 1867, 1871, 1875, 1879, 1883, 1887, 1891, 1895, 1899, 1903, 1907, 1911, 1915, 1919, 1923, 1927, 1931, 1935, 1939, 1943, 1947, 1951, 1955, 1959, 1963, 1967, 1971, 1975, 1979, 1983, 1987, 1991, 1995, 1999, 2003, 2007, 2011, 2015, 2019, 2023, 2027, 2031, 2035, 2039, 2043, 2047, 2051, 2055, 2059, 2063, 2067, 2071, 2075, 2079, 2083, 2087, 2091, 2095, 2099, 2103, 2107, 2111, 2115, 2119, 2123, 2127, 2131, 2135, 2139, 2143, 2147, 2151, 2155, 2159, 2163, 2167, 2171, 2175, 2179, 2183, 2187, 2191, 2195, 2199, 2203, 2207, 2211, 2215, 2219, 2223, 2227, 2231, 2235, 2239, 2243, 2247, 2251, 2255, 2259, 2263, 2267, 2271, 2275, 2279, 2283, 2287, 2291, 2295, 2299, 2303, 2307, 2311, 2315, 2319, 2323, 2327, 2331, 2335, 2339, 2343, 2347, 2351, 2355, 2359, 2363, 2367, 2371, 2375, 2379, 2383, 2387, 2391, 2395, 2399, 2403, 2407, 2411, 2415, 2419, 2423, 2427, 2431, 2435, 2439, 2443, 2447, 2451, 2455, 2459, 2463, 2467, 2471, 2475, 2479, 2483, 2487, 2491, 2495, 2499, 2503, 2507, 2511, 2515, 2519, 2523, 2527, 2531, 2535, 2539, 2543, 2547, 2551, 2555, 2559, 2563, 2567, 2571, 2575, 2579, 2583, 2587, 2591, 2595, 2599, 2603, 2607, 2611, 2615, 2619, 2623, 2627, 2631, 2635, 2639, 2643, 2647, 2651, 2655, 2659, 2663, 2667, 2671, 2675, 2679, 2683, 2687, 2691, 2695, 2699, 2703, 2707, 2711, 2715, 2719, 2723, 2727, 2731, 2735, 2739, 2743, 2747, 2751, 2755, 2759, 2763, 2767, 2771, 2775, 2779, 2783, 2787, 2791, 2795, 2799, 2803, 2807, 2811, 2815, 2819, 2823, 2827, 2831, 2835, 2839, 2843, 2847, 2851, 2855, 2859, 2863, 2867, 2871, 2875, 2879, 2883, 2887, 2891, 2895, 2899, 2903, 2907, 2911, 2915, 2919, 2923, 2927, 2931, 2935, 2939, 2943, 2947, 2951, 2955, 2959, 2963, 2967, 2971, 2975, 2979, 2983, 2987, 2991, 2995, 2999, 3003, 3007, 3011, 3015, 3019, 3023, 3027, 3031, 3035, 3039, 3043, 3047, 3051, 3055, 3059, 3063, 3067, 3071, 3075, 3079, 3083, 3087, 3091, 3095, 3099, 3103, 3107, 3111, 3115, 3119, 3123, 3127, 3131, 3135, 3139, 3143, 3147, 3151, 3155, 3159, 3163, 3167, 3171, 3175, 3179, 3183, 3187, 3191, 3195, 3199, 3203, 3207, 3211, 3215, 3219, 3223, 3227, 3231, 3235, 3239, 3243, 3247, 3251, 3255, 3259, 3263, 3267, 3271, 3275, 3279, 3283, 3287, 3291, 3295, 3299, 3303, 3307, 3311, 3315, 3319, 3323, 3327, 3331, 3335, 3339, 3343, 3347, 3351, 3355, 3359, 3363, 3367, 3371, 3375, 3379, 3383, 3387, 3391, 3395, 3399, 3403, 3407, 3411, 3415, 3419, 3423, 3427, 3431, 3435, 3439, 3443, 3447, 3451, 3455, 3459, 3463, 3467, 3471, 3475, 3479, 3483, 3487, 3491, 3495, 3499, 3503, 3507, 3511, 3515, 3519, 3523, 3527, 3531, 3535, 3539, 3543, 3547, 3551, 3555, 3559, 3563, 3567, 3571, 3575, 3579, 3583, 3587, 3591, 3595, 3599, 3603, 3607, 3611, 3615, 3619, 3623, 3627, 3631, 3635, 3639, 3643, 3647, 3651, 3655, 3659, 3663, 3667, 3671, 3675, 3679, 3683, 3687, 3691, 3695, 3699, 3703, 3707, 3711, 3715, 3719, 3723, 3727, 3731, 3735, 3739, 3743, 3747, 3751, 3755, 3759, 3763, 3767, 3771, 3775, 3779, 3783, 3787, 3791, 3795, 3799, 3803, 3807, 3811, 3815, 3819, 3823, 3827, 3831, 3835, 3839, 3843, 3847, 3851, 3855, 3859, 3863, 3867, 3871, 3875, 3879, 3883, 3887, 3891, 3895, 3899, 3903, 3907, 3911, 3915, 3919, 3923, 3927, 3931, 3935, 3939, 3943, 3947, 3951, 3955, 3959, 3963, 3967, 3971, 3975, 3979, 3983, 3987, 3991, 3995, 3999, 4003, 4007, 4011, 4015, 4019, 4023, 4027, 4031, 4035, 4039, 4043, 4047, 4051, 4055, 4059, 4063, 4067, 4071, 4075, 4079, 4083, 4087, 4091, 4095, 4099, 4103, 4107, 4111, 4115, 4119, 4123, 4127, 4131, 4135, 4139, 4143, 4147, 4151, 4155, 4159, 4163, 4167, 4171, 4175, 4179, 4183, 4187, 4191, 4195, 4199, 4203, 4207, 4211, 4215, 4219, 4223, 4227, 4231, 4235, 4239, 4243, 4247, 4251, 4255, 4259, 4263, 4267, 4271, 4275, 4279, 4283, 4287, 4291, 4295, 4299, 4303, 4307, 4311, 4315, 4319, 4323, 4327, 4331, 4335, 4339, 4343, 4347, 4351, 4355, 4359, 4363, 4367, 4371, 4375, 4379, 4383, 4387, 4391, 4395, 4399, 4403, 4407, 4411, 4415, 4419, 4423, 4427, 4431, 4435, 4439, 4443, 4447, 4451, 4455, 4459, 4463, 4467, 4471, 4475, 4479, 4483, 4487, 4491, 4495, 4499, 4503, 4507, 4511, 4515, 4519, 4523, 4527, 4531, 4535, 4539, 4543, 4547, 4551, 4555, 4559, 4563, 4567, 4571, 4575, 4579, 4583, 4587, 4591, 4595, 4599, 4603, 4607, 4611, 4615, 4619, 4623, 4627, 4631, 4635, 4639, 4643, 4647, 4651, 4655, 4659, 4663, 4667, 4671, 4675, 4679, 4683, 4687, 4691, 4695, 4699, 4703, 4707, 4711, 4715, 4719, 4723, 4727, 4731, 4735, 4739, 4743, 4747, 4751, 4755, 4759, 4763, 4767, 4771, 4775, 4779, 4783, 4787, 4791, 4795, 4799, 4803, 4807, 4811, 4815, 4819, 4823, 4827, 4831, 4835, 4839, 4843, 4847, 4851, 4855, 4859, 4863, 4867, 4871, 4875, 4879, 4883, 4887, 4891, 4895, 4899, 4903, 4907, 4911, 4915, 4919, 4923, 4927, 4931, 4935, 4939, 4943, 4947, 4951, 4955, 4959, 4963, 4967, 4971, 4975, 4979, 4983, 4987, 4991, 4995, 4999, 5003, 5007, 5011, 5015, 5019, 5023, 5027, 5031, 5035, 5039, 5043, 5047, 5051, 5055, 5059, 5063, 5067, 5071, 5075, 5079, 5083, 5087, 5091, 5095, 5099, 5103, 5107, 5111, 5115, 5119, 5123, 5127, 5131, 5135, 5139, 5143, 5147, 5151, 5155, 5159, 5163, 5167, 5171, 5175, 5179, 5183, 5187, 5191, 5195, 5199, 5203, 5207, 5211, 5215, 5219, 5223, 5227, 5231, 5235, 5239, 5243, 5247, 5251, 5255, 5259, 5263, 5267, 5271, 5275, 5279, 5283, 5287, 5291, 5295, 5299, 5303, 5307, 5311, 5315, 5319, 5323, 5327, 5331, 5335, 5339, 5343, 5347, 5351, 5355, 5359, 5363, 5367, 5371, 5375, 5379, 5383, 5387, 5391, 5395, 5399, 5403, 5407, 5411, 5415, 5419, 5423, 5427, 5431, 5435, 5439, 5443, 5447, 5451, 5455, 5459, 5463, 5467, 5471, 5475, 5479, 5483, 5487, 5491, 5495, 5499, 5503, 5507, 5511, 5515, 5519, 5523, 5527, 5531, 5535, 5539, 5543, 5547, 5551, 5555, 5559, 5563, 5567, 5571, 5575, 5579, 5583, 5587, 5591, 5595, 5599, 5603, 5607, 5611, 5615, 5619, 5623, 5627, 5631, 5635, 5639, 5643, 5647, 5651, 5655, 5659, 5663, 5667, 5671, 5675, 5679, 5683, 5687, 5691, 5695, 5699, 5703, 5707, 5711, 5715, 5719, 5723, 5727, 5731, 5735, 5739, 5743, 5747, 5751, 5755, 5759, 5763, 5767, 5771, 5775, 5779, 5783, 5787, 5791, 5795, 5799, 5803, 5807, 5811, 5815, 5819, 5823, 5827, 5831, 5835, 5839, 5843, 5847, 5851, 5855, 5859, 5863, 5867, 5871, 5875, 5879, 5883, 5887, 5891, 5895, 5899, 5903, 5907, 5911, 5915, 5919, 5923, 5927, 5931, 5935, 5939, 5943, 5947, 5951, 5955, 5959, 5963, 5967, 5971, 5975, 5979, 5983, 5987, 5991, 5995, 5999, 6003, 6007, 6011, 6015, 6019, 6023, 6027, 6031, 6035, 6039, 6043, 6047, 6051, 6055, 6059, 6063, 6067, 6071, 6075, 6079, 6083, 6087, 6091, 6095, 6099, 6103, 6107, 6111, 6115, 6119, 6123, 6127, 6131, 6135, 6139, 6143, 6147, 6151, 6155, 6159, 6163, 6167, 6171, 6175, 6179, 6183, 6187, 6191, 6195, 6199, 6203, 6207, 6211, 6215, 6219, 6223, 6227, 6231, 6235, 6239, 6243, 6247, 6251, 6255, 6259, 6263, 6267, 6271, 6275, 6279, 6283, 6287, 6291, 6295, 6299, 6303, 6307, 6311, 6315, 6319, 6323, 6327, 6331, 6335, 6339, 6343, 6347, 6351, 6355, 6359, 6363, 6367, 6371, 6375, 6379, 6383, 6387, 6391, 6395, 6399, 6403, 6407, 6411, 6415, 6419, 6423, 6427, 6431, 6435, 6439, 6443, 6447, 6451, 6455, 6459, 6463, 6467, 6471, 6475, 6479, 6483, 6487, 6491, 6495, 6499, 6503, 6507, 6511, 6515, 6519, 6523, 6527, 6531, 6535, 6539, 6543, 6547, 6551, 6555, 6559, 6563, 6567, 6571, 6575, 6579, 6583, 6587, 6591, 6595, 6599, 6603, 6607, 6611, 6615, 6619, 6623, 6627, 6631, 6635, 6639, 6643, 6647, 6651, 6655, 6659, 6663, 6667, 6671, 6675, 6679, 6683, 6687, 6691, 6695, 6699, 6703, 6707, 6711, 6715, 6719, 6723, 6727, 6731, 6735, 6739, 6743, 6747, 6751, 6755, 6759, 6763, 6767, 6771, 6775, 6779, 6783, 6787, 6791, 6795, 6799, 6803, 6807, 6811, 6815, 6819, 6823, 6827, 6831, 6835, 6839, 6843, 6847, 6851, 6855, 6859, 6863, 6867, 6871, 6875, 6879, 6883, 6887, 6891, 6895, 6899, 6903, 6907, 6911, 6915, 6919, 6923, 6927, 6931, 6935, 6939, 6943, 6947, 6951, 6955, 6959, 6963, 6967, 6971, 6975, 6979, 6983, 6987, 6991, 6995, 6999, 7003, 7007, 7011, 7015, 7019, 7023, 7027, 7031, 7035, 7039, 7043, 7047, 7051, 7055, 7059,