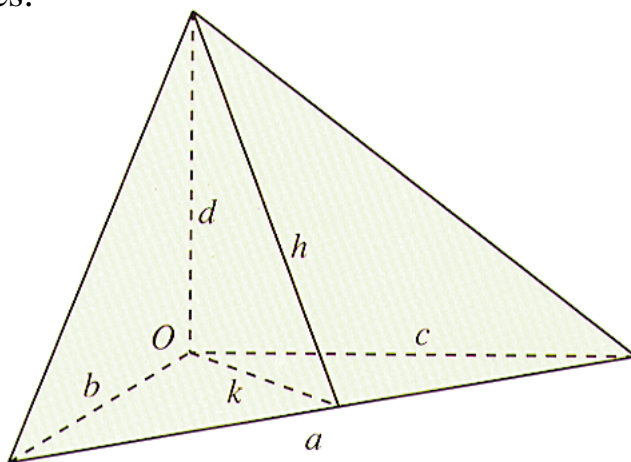


The Problem

In this right tetrahedron, the four labeled edges have measure a , b , c , and d . Vertex O is formed by the three edges that meet at right angles. Altitudes h and k are marked on two faces.

Prove that the square of the area of the face opposite vertex O equals the sum of the squares of the areas of the other three faces.



Strategies and Hints

1. The problem involves the areas of the four faces. Start by writing an expression for each of the four areas.
2. Altitude k should surprise you. Is it needed to write the area of the bottom face of the tetrahedron?
3. You might start by writing the following equation and then trying to prove it. However, you won't make much progress until you change the way the area of the bottom face is expressed!

$$\left(\frac{ah}{2}\right)^2 = \left(\frac{bc}{2}\right)^2 + \left(\frac{cd}{2}\right)^2 + \left(\frac{bd}{2}\right)^2$$