

## Are All Triangles Isosceles?

### Problem-of-the-Week

#### The Problem

What's wrong with this "proof" that every triangle is isosceles?

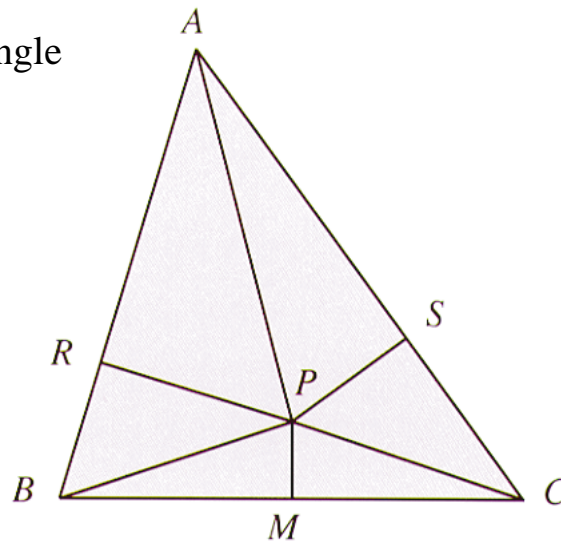
Step 1 For any triangle  $ABC$ , let the angle bisector from vertex  $A$  meet the perpendicular bisector of  $\overline{BC}$  at  $P$ .

Step 2 Drop perpendiculars from  $P$  to sides  $\overline{AB}$  and  $\overline{AC}$ , meeting these sides at points  $R$  and  $S$ .

Step 3 Since  $\angle RAP \cong \angle SAP$  and  $AP = AP$ , the right triangles  $RAP$  and  $SAP$  are congruent. Thus,  $AR = AS$  and  $RP = SP$ .

Step 4  $BP = CP$  because they are corresponding parts of the two congruent base triangles  $BPM$  and  $CPM$ . Thus, triangles  $BPR$  and  $CPS$  are congruent and  $RB = SC$ .

Step 5  $AR + RB = AS + SC$ , or  $AB = AC$  and triangle  $ABC$  is isosceles.



#### Strategies and Hints

1. The "proof" claims to show that *every* triangle is isosceles. Show that this cannot be true.
2. The figures that accompany invalid proofs are usually incorrectly drawn. Which parts of the figure above are incorrectly drawn?
3. Construct two triangles, one isosceles and one scalene. Then follow the steps in the proof to put in the rest of the segments. What is different in your two triangles?
4. Make a drawing like the figure at the right. Finish the figure by adding the points  $R$ ,  $P$ , and  $S$ . Which step in the proof does your drawing contradict?

