



The Ambiguous Case of the Law of Sines

In Lesson 7-6, you learned that you could solve a triangle using the Law of Sines if you know the measures of two angles and any side of the triangle (AAS or ASA). You can also solve a triangle by the Law of Sines if you know the measures of two sides and an angle opposite one of the sides (SSA). When you use SSA to solve a triangle, and the given angle is acute, sometimes it is possible to find two different triangles. You can use Cabri Junior to explore this case, called the **ambiguous case**, of the Law of Sines.

Step 1 Construct \overline{AB} and \overline{AC} .

- **KEYSTROKES:** F2 **ENTER**

Place the pointer anywhere on the screen and press **ENTER** to set point A. Move the pointer and press **ENTER** to set point B. Place the cursor on A and press **ENTER**, then move the pointer and press **ENTER** **CLEAR** to set point C. Then press F2 **ENTER**.

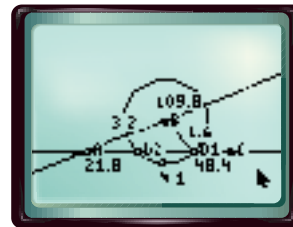
Construct a circle with a center at B so that it intersects \overline{AC} at two points.

- Place the pointer on B and press **ENTER**. Move the pointer so that the circle intersects \overline{AC} in two points and press **ENTER** **CLEAR**. Press F2 **ENTER**. Place the pointer on the circle and press **ENTER**. Then place the pointer on \overline{AC} and press **ENTER** **CLEAR**.

Step 2 Construct radius $\overline{BD_1}$ so that D_1 lies on an intersection point of circle B and \overline{AC} .

- **KEYSTROKES:** F2 **ENTER**

Place the pointer on B and press **ENTER**. Then move the pointer to D_1 and press **ENTER** **CLEAR**.



Step 3 Find the measures of $\overline{BD_1}$, \overline{AB} , \overline{AD} , $\angle A$, $\angle ABD_1$, and $\angle BD_1A$.

- **KEYSTROKES:** F5 **ENTER**

Place the pointer on a vertex and press **ENTER**. Then move the pointer to another vertex and press **ENTER**. Repeat for the other two sides. Press **CLEAR** after finding all three measures.

- **KEYSTROKES:** F5 **ENTER**

Find $m\angle A$ by placing the pointer on B and pressing **ENTER**. Move the pointer to A and press **ENTER**. Then place the pointer on D_1 and press **ENTER**. Repeat the process for the other angles. Press **CLEAR** after finding all three measures.

Geometry Software Investigation

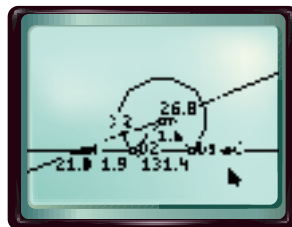
Step 4 Construct radius $\overline{BD_2}$ so that D_2 lies on the other intersection point of circle B and \overline{AC} .

• **KEYSTROKES:** F2 ▼ ▼ ENTER

Place the pointer on B and press ENTER. Then move the pointer to D_2 and press

ENTER CLEAR.

Step 5 Find the measures of $\angle ABD_2$, $\angle BD_2A$, and $\overline{AD_2}$ in $\triangle ABD_2$.



Analyze

1. Which measures are the same in both triangles?
2. Repeat the activity using different measures for $\angle A$, \overline{BD} , and \overline{AB} . How do the results compare to the earlier results?

Make a Conjecture

3. Compare your results with those of your classmates. How do the results compare?
4. What would have to be true about circle B in order for there to be one unique solution? Test your conjecture by repeating the activity.
5. Is it possible, given the measures of \overline{BD} , \overline{AB} , and $\angle A$, to have no solution? Test your conjecture and explain.