

A TEACHER REFLECTS



I was watching two of my students, Carmen and Natasha, try to make 5,225,000 using 12 beads on their Chinese abaci. Both had made 5,225 with 12 beads, sharing the same strategy: make the number first using the fewest beads, then make substitutions until you find a combination that uses the given number of beads. But Natasha was now struggling to figure out simply how to make 5,225,000 using the fewest beads.

“How did you make it?” Natasha asked, peering over at Carmen’s abacus. “Like this.” Carmen showed her a correct representation of 5,225,000. “My beads are in different places,” Natasha sighed. “How come if we made the same number, they look different?” Carmen studied Natasha’s abacus. “I think your number’s too small,” Carmen said. “You’ve made five million two hundred twenty-five.” Natasha just looked even more puzzled. Carmen sighed. “Let me see if I can explain it.”

Carmen put a piece of paper under her abacus. “See, my number is like this,” she began, and wrote each digit lined up at the bottom of each column. “It’s 5 million, two hundred twenty-five thousand, because my number ends in the thousands place.” Natasha looked bewildered. “But that’s what I have!” she said, pointing to the 5,225,000 that Carmen had written. Carmen shook her head. “Natasha, your zeros are in a different place.” I was surprised at the extent of Natasha’s confusion. She had had no difficulty using smaller numbers on the abacus, but now that the numbers were larger, she seemed lost.

Carmen’s face suddenly lit up. “Wait!” she exclaimed. “I have an idea!” Carmen positioned her abacus between them, and cleared it. “Try this. What’s this?” She pushed up a ten bead and a one bead. “11. So?” Natasha said, sounding mildly insulted. “Okay, what’s this?” Carmen went on. She pushed up a one-hundred bead and a one bead. “101,” Natasha replied, unimpressed. “So what’s the difference?” Carmen was excited. I could see what she was driving at, and made a mental note about Carmen’s insight. “101’s bigger.” Natasha tried. Clearly, this was not the answer Carmen had wanted. She sighed. “No, I mean how do you know it’s 101?” “Because there’s a bead for one hundred and a bead for one,” Natasha replied. “And where’s the zero?” Carmen went on. Natasha looked at the abacus. “It’s...wait...there isn’t one. Well, no...there is, but there’s no beads there.” She indicated the tens column. A look of realization started growing. “Ohhhhh...”

Carmen grabbed Natasha’s abacus. “So this is what you made,” she said, starting to make the number. Natasha broke in. “No, let me try it. I had this.” She made 5,000,225, and looked at it for a moment. “So these are all zeros!” she exclaimed, pointing to the three empty columns.