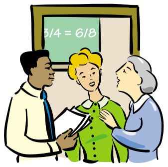
A TEACHER REFLECTS



Phase Three was the first place my students saw a zero exponent in the curriculum. It was also one of the first times they encountered a mathematical convention where familiar symbols are used in a counter-intuitive way; i.e., zero as an exponent has nothing to do with what students understand about the number 0. I decided to spend some time with the class discussing why a nonzero number with a 0 exponent equals 1, rather than 0. The exception is 0^0. I began by showing them patterns of descending powers in various bases. By repeated division, any nonzero number to the zero power always equals 1.

3^4	= 81	10^4	= 10,000	2^4	= 16
3^3	= 27	10^3	= 1,000	2^3	= 8
3^2	= 9	10^2	= 100	2^2	= 4
3^1	= 3	10^1	= 10	2^1	= 2
3^0	= 1	10^0	= 1	2^0	= 1

After looking at these lists, my students wanted to create a rule that said, "Any number to the zero power = 1." To check their idea, they asked me about numbers that they considered outrageous, so I got questions like, "You mean 3,492,112,563,498 to the zero power = 1?"

I knew that the actual rule states that any nonzero number to the zero power = 1, so I decided to help them think about this. I posed the question "What would happen if we added 0 to these lists?" Thinking about 0^4, 0^3, 0^2, and 0^1 made my students stop and think for a moment, but they quickly concluded that these would all equal 0, because they could multiply 0 times itself. When we got to 0^0, the class was divided on whether they thought it should equal 1 or 0. They didn't want to accept my answer that it was neither, that it was "undefined," because they couldn't really understand what that meant. I eventually made this chart for the class.

4 ÷ 4	1
4 ÷ 3	1.3333333...
4 ÷ 2	2
4 ÷ 1	4
4 ÷ 0.5	8
4 ÷ 0.25	16
4 ÷ 0.10	40
4 ÷ 0.05	80
4 ÷ 0.0000005	8,000,000

Most students could see that the closer to 0 the divisor got, the larger the quotient would get, until it would be too huge to measure. Some insisted that dividing by 0 would equal infinity. I pointed out that infinity is not a number, and that was why dividing by 0 was undefined, or impossible. We concluded by amending the rule to say that any whole number that was not zero, raised to the zero power, would = 1.