

A TEACHER REFLECTS



Folds and Regions

Students were very enthusiastic about exploring the paper-folding problem. Before they began experimenting and setting up their tables, I asked students to make some “educated guesses” about additional questions.

- How many times do you think you would need to fold the sheet of paper in half to have at least 1,000 regions?
- How many times do you think you would need to fold the sheet of paper in half to have at least 1,000,000 regions?
- After any number of folds, do you think the last digit in the number of regions will ever be a 7? Why or why not?

After students had a chance to think about these questions, I had them make tables and complete the investigation. Later, we revisited the tables. I asked volunteers to describe any patterns they saw in the tables. Danielle began by pointing out that, except for zero folds, the number of regions is always an even number. In fact, she pointed out, the number of regions always ends in a 2, 4, 6, or 8. Jerome took this one step further. He said, “The pattern is 2, 4, 8, 6. It just keeps repeating.”

This led me to pose the following question. I asked the class if they could predict the last digit in the number of regions when the sheet of paper is folded 103 times. I had students work in

groups for this investigation and then asked for a volunteer to share his or her group’s reasoning. Teresa described her group’s results this way: “We noticed that whenever the number of folds is a multiple of 4, the number of regions ends in a 6. So, we figured that 100 folds would also give a number of regions ending in a 6. Then the number of regions for 101 folds must end in a 2, the number of regions for 102 folds must end in a 4, and the number of regions for 103 folds must end in an 8.”

Number of Folds	Number of Regions	Number of Regions (as a power of 2)
0	1	2^0
1	2	2^1
2	4	2^2
3	8	2^3
4	16	2^4
5	32	2^5
6	64	2^6
7	128	2^7
8	256	2^8
9	512	2^9
10	1,024	2^{10}
11	2,048	2^{11}
12	4,096	2^{12}
13	8,192	2^{13}
14	16,384	2^{14}
15	32,768	2^{15}
16	65,536	2^{16}
17	131,072	2^{17}
18	262,144	2^{18}
19	524,288	2^{19}
20	1,048,576	2^{20}
21	2,097,152	2^{21}
22	4,194,304	2^{22}
23	8,388,608	2^{23}
24	16,777,216	2^{24}
25	33,554,432	2^{25}