

9-5

Direct Variation

MAIN IDEA

Use direct variation to solve problems.

New Vocabulary

direct variation
constant of variation

Math Online

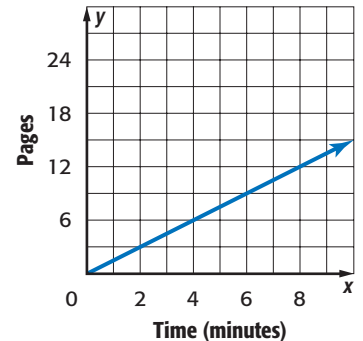
glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz

▶ GET READY for the Lesson

COMPUTERS Use the graph at the right that shows the output of a color printer.

1. What is the constant rate of change, or slope, of the line?
2. Is the total number of pages printed always proportional to the printing time? If so, what is the constant ratio?
3. Compare the constant rate of change to the constant ratio.



In the example above, the number of minutes and the number of pages printed both vary, while the ratio of pages printed to minutes, 1.5 pages per minute, remains constant.

When the ratio of two variable quantities is constant, their relationship is called a **direct variation**. The constant ratio is called the **constant of variation**.

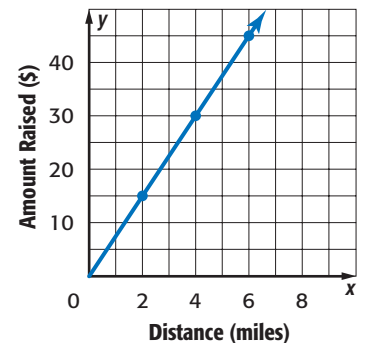


Real-World EXAMPLE

Find a Constant Ratio

- 1 **FUNDRAISER** The amount of money Robin has raised for a bike-a-thon is shown in the graph at the right. Determine the amount that Robin raises for each mile she rides.

Since the graph of the data forms a line, the rate of change is constant. Use the graph to find the constant ratio.



$$\frac{\text{amount raised}}{\text{distance}} \rightarrow \frac{15}{2} \text{ or } \frac{7.5}{1} \quad \frac{30}{4} \text{ or } \frac{7.5}{1} \quad \frac{45}{6} \text{ or } \frac{7.5}{1} \quad \frac{60}{8} \text{ or } \frac{7.5}{1}$$

Robin raises \$7.50 for each mile she rides.



CHECK Your Progress

- SKYDIVING** Two minutes after a skydiver opens his parachute, he has descended 1,900 feet. After 5 minutes, he has descended 4,750 feet. If the distance varies directly as the time, at what rate is the skydiver descending?

In a direct variation equation, the constant rate of change, or slope, is assigned a special variable, k .

Direct Variation

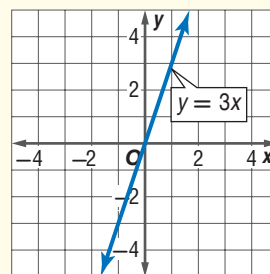
Key Concept

Words A direct variation is a relationship in which the ratio of y to x is a constant, k . We say y varies directly with x .

Symbols $k = \frac{y}{x}$ or $y = kx$,
where $k \neq 0$

Example $y = 3x$

Model



Real-World Link

Most pets age at a different rate than their human companions. For example, a 3-year-old dog is often considered to be 21 in human years.

2 Real-World EXAMPLE Solve a Direct Variation

2 PETS Refer to the information at the left. Assume that the age of a dog varies directly as its equivalent age in human years. What is the human-year age of a dog that is 6 years old?

Write an equation of direct variation. Let x represent the dog's actual age and let y represent the human-equivalent age.

$$\begin{aligned} y &= kx && \text{Direct variation} \\ 21 &= k(3) && y = 21, x = 3 \\ 7 &= k && \text{Simplify.} \\ y &= 7x && \text{Substitute for } k = 7. \end{aligned}$$

Use the equation to find y when $x = 6$.

$$\begin{aligned} y &= 7x \\ y &= 7(6) && x = 6 \\ y &= 42 && \text{Multiply.} \end{aligned}$$

A dog that is 6 years old is 42 years old in human-equivalent years.

✓ CHECK Your Progress

b. **SHOPPING** A grocery store sells 6 oranges for \$2. How much would it cost to buy 10 oranges? Round to the nearest cent if necessary.

Study Tip

Proportions
In Example 2, you can also use a proportion to solve direct variation problems. Write ratios comparing the human equivalent age to the actual age.

$$\begin{aligned} \frac{21}{3} &= \frac{x}{6} \\ 126 &= 3x \\ 42 &= x \end{aligned}$$

In a direct variation, the constant of variation k is a constant rate of change. When the x -value changes by an amount a , then the y -value will change by the corresponding amount ka . In the previous example, when x changed by a factor of 6, y changed by 7(6) or 42.

Study Tip

Look Back
To review proportional relationships, see Lessons 4-2 and 4-5.

Not all relationships with a constant rate of change are proportional. Likewise, not all linear functions are direct variations.

EXAMPLES Identify Direct Variation

Determine whether each linear function is a direct variation. If so, state the constant of variation.

3

Miles, x	25	50	75	100
Gallons, y	1	2	3	4

Compare the ratios to check for a common ratio.

$$\frac{\text{gallons}}{\text{miles}} \rightarrow \frac{1}{25} \quad \frac{2}{50} \text{ or } \frac{1}{25} \quad \frac{3}{75} \text{ or } \frac{1}{25} \quad \frac{4}{100} \text{ or } \frac{1}{25}$$

Since the ratios are the same, the function is a direct variation. The constant of variation is $\frac{1}{25}$.

4

Hours, x	2	4	6	8
Earnings, y	36	52	68	84

$$\frac{\text{earnings}}{\text{hours}} \rightarrow \frac{36}{2} \text{ or } \frac{18}{1} \quad \frac{52}{4} \text{ or } \frac{13}{1} \quad \frac{68}{6} \text{ or } \frac{11.33}{1} \quad \frac{84}{8} \text{ or } \frac{10.50}{1}$$

The ratios are not the same, so the function is not a direct variation.

CHECK Your Progress

c.

Days, x	5	10	15	20
Height, y	12.5	25	37.5	50

d.

Time, x	4	6	8	10
Distance, y	12	16	20	24

Study Tip

Direct Variations
Notice that the graph of a direct variation, which is a proportional linear relationship, is a line that passes through the origin.

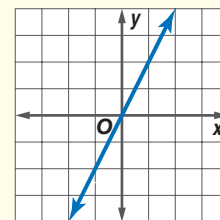
Proportional Linear Function

Concept Summary

Table

x	-2	-1	1	2
y	-4	-2	2	4
$\frac{y}{x}$	2	2	2	2

Graph



Equation

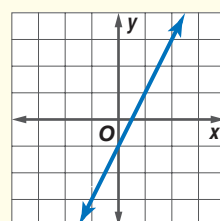
$$y = 2x$$

Nonproportional Linear Function

Table

x	-2	-1	1	2
y	-5	-3	1	3
$\frac{y}{x}$	$\frac{5}{2}$	3	1	$\frac{3}{2}$

Graph



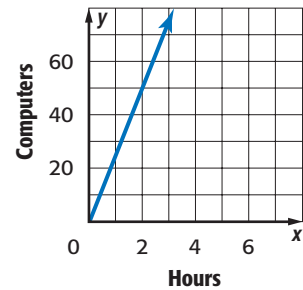
Equation

$$y = 2x - 1$$

CHECK Your Understanding

Example 1
(p. 487)

- MANUFACTURING** The number of computers built varies directly as the number of hours the production line operates. What is the ratio of computers built to hours of production?



Example 2
(p. 488)

- TRANSPORTATION** A charter bus travels 210 miles in $3\frac{1}{2}$ hours. Assuming that the distance traveled is directly proportional to the time traveled, how far will the bus travel in 6 hours?

Examples 3, 4
(p. 489)

- Determine whether the linear function is a direct variation. If so, state the constant of variation.

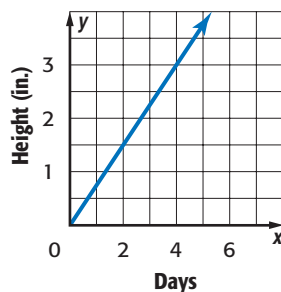
Hours, x	2	3	4	5
Miles, y	116	174	232	290

Practice and Problem Solving

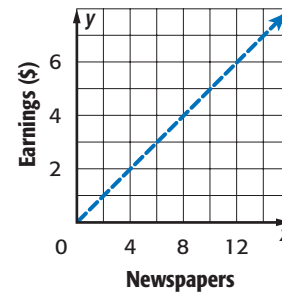
HOMework HELP

For Exercises	See Examples
4–5	1
6–11	2
12–15	3, 4

- GARDENING** Janelle planted ornamental grass seeds. After the grass breaks the soil surface, its height varies directly with the number of days. What is the rate of growth?



- JOBS** The amount Dusty earns is directly proportional to the number of newspapers he delivers. How much does Dusty earn for each newspaper delivery?



- SUBMARINES** Ten minutes after a submarine is launched from a research ship, it is 25 meters below the surface. After 30 minutes, the submarine has descended 75 meters. At what rate is the submarine diving?
- MOVIES** The Stratton family rented 3 DVDs for \$10.47. The next weekend, they rented 5 DVDs for \$17.45. What is the rental fee for a DVD?
- MEASUREMENT** Morgan used 3 gallons of paint to cover 1,050 square feet and 5 gallons to paint an additional 1,750 square feet. How many gallons of paint would she need to cover 2,800 square feet?
- MEASUREMENT** The weight of an object on Mars varies directly with its weight on Earth. An object that weighs 70 pounds on Mars weighs 210 pounds on Earth. If an object weighs 160 pounds on Earth, how much would it weigh on Mars?



Real-World Link . . .

The aspect ratio of a television screen describes the ratio of the width of the screen to the height. Standard screens have an aspect ratio of 4:3 while wide-screen televisions have an aspect ratio of 16:9.

10. **ELECTRONICS** The height of a wide-screen television screen is directly proportional to its width. A manufacturer makes a television screen that is 60 centimeters wide and 33.75 centimeters high. Find the height of a television screen that is 90 centimeters wide.
11. **BAKING** A cake recipe requires $2\frac{3}{4}$ cups of flour for 12 servings. How much flour is required to make a cake that serves 30?

Determine whether each linear function is a direct variation. If so, state the constant of variation.

12.

Pictures, x	5	6	7	8
Profit, y	20	24	28	32

13.

Minutes, x	200	400	600	800
Cost, y	65	115	165	215

14.

Age, x	10	11	12	13
Grade, y	5	6	7	8

15.

Price, x	10	15	20	25
Tax, y	0.70	1.05	1.40	1.75

ALGEBRA If y varies directly with x , write an equation for the direct variation. Then find each value.

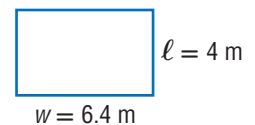
16. If $y = -12$ when $x = 9$, find y when $x = -4$.
17. Find y when $x = 10$ if $y = 8$ when $x = 20$.
18. If $y = -6$ when $x = -14$, what is the value of x when $y = -4$?
19. Find x when $y = 25$, if $y = 7$ when $x = 8$.
20. Find y when $x = 5$, if $y = 12.6$ when $x = 14$.
21. **MEASUREMENT** The number of centimeters in a measure varies directly as the number of inches. Find the measure of an object in centimeters if it is 50 inches long.

Inches, x	6	9	12	15
Centimeters, y	15.24	22.86	30.48	38.10

EXTRA PRACTICE

See pages 692, 708.

22. **MEASUREMENT** The length of the rectangle shown varies directly as its width. What is the perimeter of a rectangle that is 10 meters long?



H.O.T. Problems

23. **OPEN ENDED** Identify values for x and y in a direct variation relationship where $y = 9$ when $x = 16$.
24. **CHALLENGE** The amount of stain needed to cover a wood surface is directly proportional to the area of the surface. If 3 pints are required to cover a square deck with a side of 7 feet, how many pints of stain are needed to paint a square deck with a side of 10 feet 6 inches?
25. **WRITING IN MATH** Write a direct variation equation. Then triple the x -value and explain how to find the corresponding change in the y -value.

26. Students in a science class recorded lengths of a stretched spring, as shown in the table below.

Length of Stretched Spring	
Distance Stretched, x (centimeters)	Mass, y (grams)
0	0
2	12
5	30
9	54
12	72

Which equation best represents the relationship between the distance stretched x and the mass of an object on the spring y ?

- A $y = -6x$ C $y = -\frac{x}{6}$
 B $y = 6x$ D $y = \frac{x}{6}$

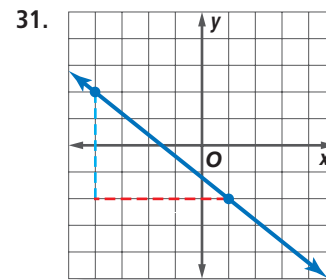
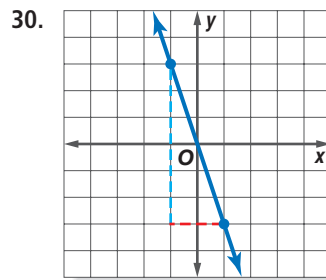
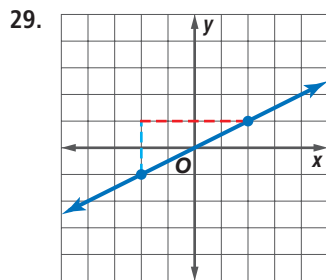
27. **SHORT RESPONSE** Nicole read 24 pages during a 30-minute independent reading period. How many pages would she read in 45 minutes?

28. To make fruit punch, Kelli must add 8 ounces of pineapple juice for every 12 ounces of orange juice. If she uses 32 ounces of orange juice, which proportion can she use to find x , the number of ounces of pineapple juice she should add to make the punch?

- F $\frac{8}{12} = \frac{32}{x}$ H $\frac{8}{12} = \frac{x}{32}$
 G $\frac{8}{x} = \frac{32}{12}$ J $\frac{x}{12} = \frac{8}{32}$

Spiral Review

Find the slope of each line. (Lesson 9-4)



32. **JOBS** The function $p = 7.5h$ describes the relationship between the number of hours h Callie works and the amount she is paid p . Graph the function. Then use your graph to determine how much Callie can expect to earn if she works 20 hours. (Lesson 9-3)
33. **HEALTH** Many health authorities recommend that a healthy diet contains no more than 30% of its Calories from fat. If Jennie consumes 1,500 Calories each day, what is the maximum number of Calories she should consume from fat? (Lesson 5-3)

▶ GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. (Lesson 1-9)

34. $7 + a = 15$ 35. $23 = d + 44$ 36. $28 = n - 14$ 37. $t - 22 = -31$