

6-3

Rate of Change and Slope

MAIN IDEA

Identify rate of change and slope using tables and graphs.

New Vocabulary

rate of change
slope

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz

▶ GET READY for the Lesson

HEIGHTS The table shows Stephanie's height at ages 9 and 12.

Age (yr)	9	12
Height (in.)	53	59

1. What is the change in Stephanie's height from ages 9 to 12?
2. Over what number of years did this change take place?
3. Write a rate that compares the change in Stephanie's height to the change in age. Express your answer as a unit rate and explain its meaning.

A **rate of change** is a rate that describes how one quantity changes in relation to another. A rate of change is usually expressed as a unit rate.

EXAMPLE Find Rate of Change from a Table

- 1 FUNDRAISING** The table shows the amount of money a Booster Club made washing cars for a fundraiser. Use the information to find the rate of change in dollars per car.

Cars Washed	
Number	Money (\$)
5	40
10	80
15	120
20	160

+5 ↘ ↘ +40
+5 ↘ ↘ +40
+5 ↘ ↘ +40

Find the unit rate to determine the rate of change.

$$\frac{\text{change in money}}{\text{change in cars}} = \frac{40 \text{ dollars}}{5 \text{ cars}} \quad \text{The money earned increases by } \$40 \text{ for every 5 cars.}$$

$$= \frac{8 \text{ dollar}}{1 \text{ car}} \quad \text{Write as a unit rate.}$$

So, the number of dollars earned increases by \$8 for every car washed.

✓ CHECK Your Progress

- a. **PLANES** The table shows the number of miles a plane traveled while in flight. Use the information to find the approximate rate of change in miles per minute.

Time (min)	30	60	90	120
Distance (mi)	290	580	870	1,160

EXAMPLE**Find Rate of Change from a Graph****Reading Math**

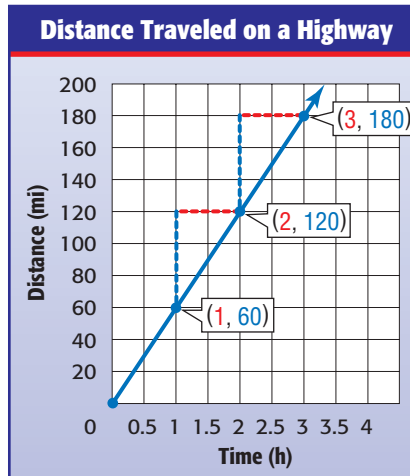
Ordered Pairs The ordered pair (2, 120) represents traveling 120 miles in 2 hours.

- 2 DRIVING** The graph represents the distance traveled while driving on a highway. Use the graph to find the rate of change in miles per hour.

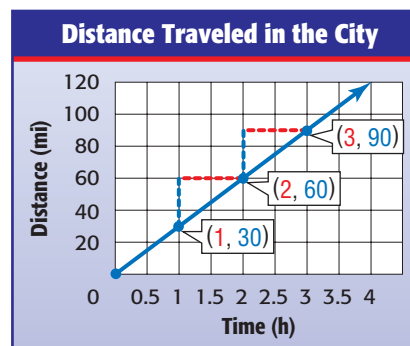
To find the rate of change, pick any two points on the line, such as (1, 60) and (2, 120).

$$\begin{aligned} \frac{\text{change in miles}}{\text{change in hours}} &= \frac{(120 - 60) \text{ miles}}{(2 - 1) \text{ hours}} \\ &= \frac{60 \text{ miles}}{1 \text{ hour}} \end{aligned}$$

The distance increases by 60 miles in 1 hour. So, the rate of traveling on a highway is 60 miles per hour.

**✓ CHECK Your Progress**

- b. **DRIVING** Use the graph to find the rate of change in miles per hour while driving in the city.



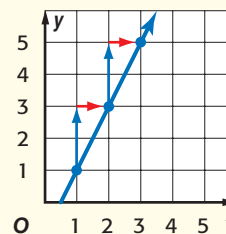
Notice that the graph in Example 2 about driving on a highway represents a rate of change of 60 mph. The graph in Check Your Progress about driving in the city is not as steep. It represents a rate of change of 30 mph.

The constant rate of change in y with respect to the constant change x is also called the slope of a line. Slope is a number that tells how steep the line is. The slope is the same for any two points on a straight line.

Slope**Key Concept**

Slope is the rate of change between any two points on a line.

$$\begin{aligned} \text{slope} &= \frac{\text{change in } y}{\text{change in } x} \quad \leftarrow \begin{array}{l} \text{vertical change} \\ \text{horizontal change} \end{array} \\ &= \frac{2}{1} \text{ or } 2 \end{aligned}$$





Real-World EXAMPLE

Find Slope



Real-World Link

Lightning strikes somewhere on the surface of Earth about 100 times every second.

Source: National Geographic

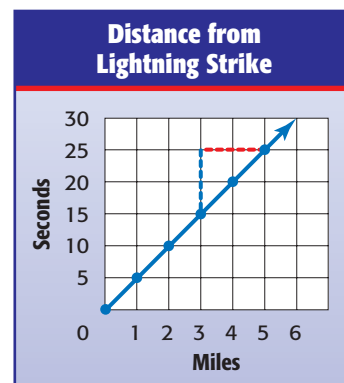
PHYSICAL SCIENCE The table below shows the relationship between the number of seconds y it takes to hear the thunder after a lightning strike and the distance x you are from the lightning.

Distance (x)	0	1	2	3	4	5
Seconds (y)	0	5	10	15	20	25

- 3 Graph the data. Then find the slope of the line. Explain what the slope represents.

$$\begin{aligned} \text{slope} &= \frac{\text{change in } y}{\text{change in } x} && \text{Definition of slope} \\ &= \frac{25 - 10}{5 - 2} && \text{Use } (2, 10) \text{ and } (5, 25). \\ &= \frac{15}{3} && \begin{array}{l} \leftarrow \text{seconds} \\ \leftarrow \text{miles} \end{array} \\ &= \frac{5}{1} && \text{Simplify.} \end{aligned}$$

So, for every 5 seconds between a lightning flash and the sound of the thunder, there is 1 mile between you and the lightning strike.



CHECK Your Progress

- c. **WATER** Graph the data. Then find the slope of the line. Explain what the slope represents.

Water Level Loss

Week	Water Loss (cm)
1	1.5
2	3
3	4.5
4	6



CHECK Your Understanding

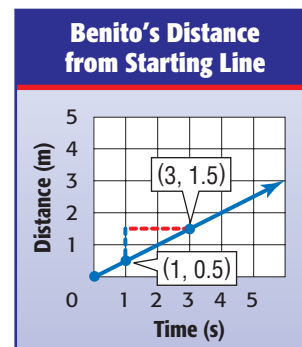
Example 1
(p. 293)

1. Use the information in the table to find the rate of change in degrees per hour.

Temperature (°F)	54	57	60	63
Time	6 A.M.	8 A.M.	10 A.M.	12 P.M.

Example 2
(p. 294)

2. **DISTANCE** The graph shows Benito's distance from the starting line. Use the graph to find the rate of change.



Example 3
(p. 295)

3. **SNACKS** The table below shows the number of small packs of fruit snacks y per box x . Graph the data. Then find the slope of the line. Explain what the slope represents.

Boxes (x)	3	5	7	9
Packs (y)	24	40	56	72

Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
4–6	1
7, 8	2
9, 10	3

For Exercises 4 and 5, find the rate of change for each table.

4.

Time (s)	Distance (m)
0	6
1	12
2	18
3	24

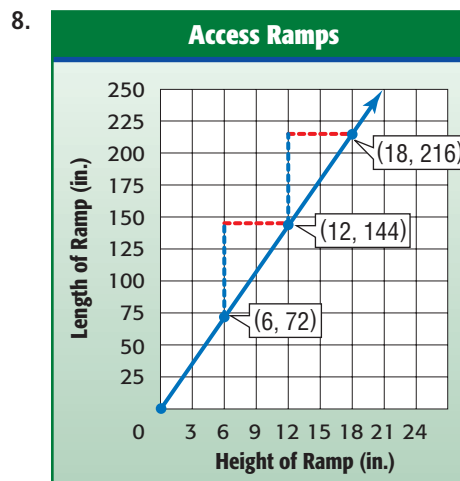
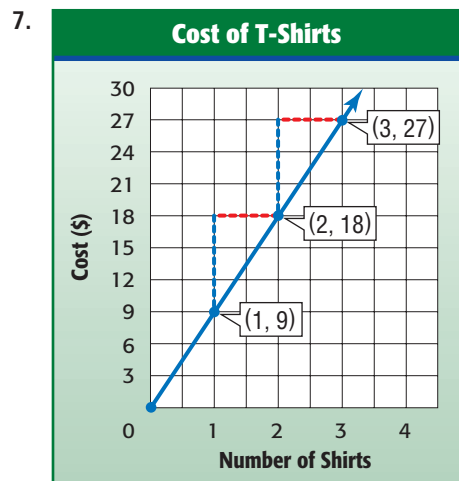
5.

Time (h)	Wage (\$)
0	0
1	9
2	18
3	27

6. The number of minutes included in different cell phone plans and the costs are shown in the table. What is the approximate rate of change in cost per minute?

Cost (\$)	38	50	62	74	86
Minutes	1,000	1,500	2,000	2,500	3,000

For Exercises 7 and 8, find the rate of change for each graph.



9. **CYCLING** The table shows the distance y Cheryl traveled in x minutes while competing in the cycling portion of a triathlon. Graph the data. Then find the slope of the line. Explain what the slope represents.

EXTRA PRACTICE

See pages 682, 709.

Time (min)	45	90	135	180
Distance (km)	5	10	15	20

10. **MAPS** The table shows the key for a map. Graph the data. Then find the slope of the line.

Distance on Map (in.)	2	4	6	8
Actual Distance (mi)	40	80	120	160

11. **WATER** At 1:00, the water level in a pool is 13 inches. At 2:30, the water level is 28 inches. What is the rate of change?
12. **MONEY** Dwayne opens a savings account with \$75. He makes the same deposit every month and makes no withdrawals. After 3 months, he has \$150. After 6 months, he has \$300. After 9 months, he has \$450 dollars. What is the rate of change?

H.O.T. Problems

13. **OPEN ENDED** Make a table where the rate of change is 6 inches for every foot.
14. **WRITING IN MATH** Write a problem to represent a rate of change of \$15 per item.

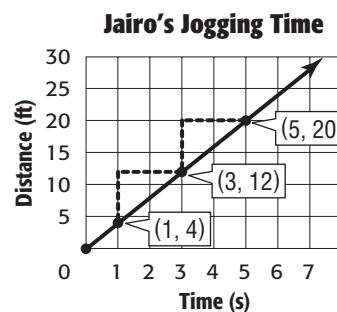
TEST PRACTICE

15. Use the information in the table to find the rate of change.

Number of Apples	Number of Seeds
3	30
7	70
11	110

- A $\frac{10}{1}$ C $\frac{40}{4}$
 B $\frac{1}{10}$ D $\frac{4}{40}$

16. **SHORT RESPONSE** Find the slope of the line below that shows the distance Jairo traveled while jogging.



Spiral Review

17. **GROCERIES** Three pounds of pears cost \$3.57. At this rate, how much would 10 pounds cost? (Lesson 6-2)

Write each ratio as a fraction in simplest form. (Lesson 6-1)

18. 9 feet in 21 minutes 19. 36 calls in 2 hours 20. 14 SUVs out of 56 vehicles

GET READY for the Next Lesson

PREREQUISITE SKILL Solve. (Page 674)

21. 2.5×20 22. 3.5×4 23. $104 \div 16$ 24. $4,200 \div 2,000$