Follow the steps below to discover how the triangles at the right are related.

**Mini Lab**

1. **Copy both triangles onto tracing paper.**
   - **Step 1**

2. **Measure and record the sides of each triangle.**
   - **Step 2**

3. **Cut out both triangles.**
   - **Step 3**

1. **Compare the angles of the triangles by matching them up.**
   - Identify the angle pairs that have equal measure.
2. **Express the ratios \( \frac{DF}{LK} \), \( \frac{EF}{JK} \), and \( \frac{DE}{LJ} \) as decimals to the nearest tenth.
3. **What do you notice about the ratios of these sides of matching triangles?**

A **polygon** consists of a sequence of consecutive line segments in a plane, placed end to end to form a simple closed figure. Polygons that have the same shape are called **similar** polygons. In the figure below, polygon \( ABCD \) is similar to polygon \( WXYZ \). This is written as polygon \( ABCD \sim \) polygon \( WXYZ \).

The parts of similar figures that “match” are called **corresponding parts**.
The similar triangles in the Mini Lab suggest the following.

**Similar Polygons**

**Words**
If two polygons are similar, then
- their corresponding angles are **congruent**, or have the same measure, and
- the measures of their corresponding sides are proportional.

**Model**

\[
\triangle ABC \sim \triangle XYZ
\]

**Symbols**
\[
\angle A \cong \angle X, \angle B \cong \angle Y, \angle C \cong \angle Z, \text{ and } \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}
\]

**EXAMPLE**

**Identify Similar Polygons**

1. Determine whether rectangle **HJKL** is similar to rectangle **MNPQ**. Explain.

First, check to see if corresponding angles are congruent.

Since the two polygons are rectangles, all of their angles are right angles. Therefore, all corresponding angles are congruent.

Next, check to see if corresponding sides are proportional.

\[
\frac{HJ}{MN} = \frac{7}{10}, \quad \frac{JK}{NP} = \frac{3}{6} \quad \text{or} \quad \frac{1}{2}, \quad \frac{KL}{PQ} = \frac{7}{10}, \quad \frac{LH}{QM} = \frac{3}{6} \quad \text{or} \quad \frac{1}{2}
\]

Since \(\frac{7}{10}\) and \(\frac{1}{2}\) are not equivalent ratios, rectangle **HJKL** is not similar to rectangle **MNPQ**.

**Check Your Progress**

Determine whether these polygons are similar. Explain.

a.  
   - The ratio of the lengths of two corresponding sides of two similar polygons is called the **scale factor**. You can use the scale factor of similar figures or a proportion to find missing measures.
**EXAMPLE**

Find Missing Measures

**GEOMETRY**  Given that polygon $WXYZ \sim$ polygon $ABCD$, find the missing measure.

**METHOD 1**

**Write a proportion.**

The missing measure $m$ is the length of $XY$. Write a proportion.

\[
\frac{XY}{BC} = \frac{YZ}{CD}
\]

\[
\frac{m}{12} = \frac{15}{10}
\]

$XY = m$, $BC = 12$, $YZ = 15$, and $CD = 10$.

Find the cross products.

$10m = 12 \cdot 15$  Multiply.

$m = 18$  Divide each side by 10.

**METHOD 2**

**Use the scale factor to write an equation.**

Find the scale factor from polygon $WXYZ$ to polygon $ABCD$.

scale factor: $\frac{YZ}{CD} = \frac{15}{10}$  or $\frac{3}{2}$

The scale factor is the constant of proportionality.

\[
\frac{XY}{BC} = \frac{YZ}{CD}
\]

\[
\frac{m}{12} = \frac{15}{10}
\]

Let $m$ represent the measure of $XY$.

\[
m = \frac{3}{2} \cdot 12
\]

\[
m = 18
\]

**C H O O S E Your Method**

Find each missing measure above.

c.  $WZ$

d.  $AB$

Square A $\sim$ square B with a scale factor of 3:2. Notice that the ratio of their perimeters is 12:8 or 3:2.

<table>
<thead>
<tr>
<th>Square</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12 m</td>
</tr>
<tr>
<td>B</td>
<td>8 m</td>
</tr>
</tbody>
</table>
This and other related examples suggest the following.

### Ratios of Similar Figures

**Words** If two figures are similar with a scale factor of \( \frac{a}{b} \), then the perimeters of the figures have a ratio of \( \frac{a}{b} \).

**Model**

![Diagram showing two similar triangles with a scale factor of \( \frac{a}{b} \).](image)

### Test Example

Triangle \( \triangle LMN \) is similar to triangle \( \triangle PQR \). If the perimeter of \( \triangle LMN \) is 64 units, what is the perimeter of \( \triangle PQR \)?

- A 108 units
- B 96 units
- C 48 units
- D 36 units

**Read the Item**

You know the measures of two corresponding sides and the perimeter of \( \triangle LMN \). You need to find the perimeter of \( \triangle PQR \).

**Solve the Item**

Triangle \( \triangle LMN \sim \triangle PQR \) with a scale factor of \( \frac{24}{18} \) or \( \frac{4}{3} \). The ratio of the perimeters of \( \triangle LMN \) to \( \triangle PQR \) is also \( \frac{4}{3} \).

\[
\frac{\text{perimeter of } \triangle LMN}{\text{perimeter of } \triangle PQR} = \frac{64}{x} = \frac{4}{3}
\]

Scale factor relating \( \triangle LMN \) to \( \triangle PQR \)

\[
64 \cdot 3 = 4 \cdot x
\]

Find the cross products.

\[
192 = 4x
\]

Multiply.

\[
\frac{192}{4} = \frac{4x}{4}
\]

Divide each side by 4.

\[
48 = x
\]

Simplify.

The answer is C.

### Check Your Progress

e. Square \( \square KLMN \) is similar to square \( \square TUVW \). If the perimeter of square \( \square KLMN \) is 32 units, what is the perimeter of square \( \square TUVW \)?

- F 128 units
- G 96 units
- H 64 units
- J 40 units
Example 1  
(p. 219)

Determine whether each pair of polygons is similar. Explain.

1. \[
\begin{align*}
&3 \quad 5 \\
&4 \quad 12
\end{align*}
\]

2. \[
\begin{align*}
&8 \quad 10 \\
&6 \quad 7.5
\end{align*}
\]

Example 2  
(p. 220)

3. In the figure at the right, \(\triangle FGH \sim \triangle KLM\). Write and solve a proportion to find each missing side measure.

\[
\begin{align*}
&F \\
&6 \\
&9 \\
&x
\end{align*}
\]

Example 3  
(p. 221)

4. **MULTIPLE CHOICE** \(\triangle ABC\) is similar to \(\triangle XYZ\).
   If the perimeter of \(\triangle ABC\) is 40 units, what is the perimeter of \(\triangle XYZ\)?
   
   \[
   \begin{align*}
   &A \quad \text{10 units} \quad C \quad \text{40 units} \\
   &B \quad \text{20 units} \quad D \quad \text{80 units}
   \end{align*}
   \]

---

**Practice and Problem Solving**

Determine whether each pair of polygons is similar. Explain.

5. \[
\begin{align*}
&3 \quad 4 \\
&7 \quad 8
\end{align*}
\]

6. \[
\begin{align*}
&3 \quad 3 \\
&5 \quad 5
\end{align*}
\]

7. \[
\begin{align*}
&20 \quad 16 \\
&24 \quad 12
\end{align*}
\]

8. \[
\begin{align*}
&5 \quad 8 \\
&6 \quad 6
\end{align*}
\]

Each pair of polygons is similar. Write and solve a proportion to find each missing side measure.

9. \[
\begin{align*}
&12 \quad x \\
&8 \quad 3
\end{align*}
\]

10. \[
\begin{align*}
&5 \quad x \\
&10 \quad 4.8
\end{align*}
\]

11. \[
\begin{align*}
&29 \quad x \\
&21 \quad 10.5
\end{align*}
\]

12. \[
\begin{align*}
&22.4 \quad 12 \\
&8 \quad x
\end{align*}
\]
13. **LIFE SCIENCE**  The scale factor from the model of a human inner ear to the actual ear is 55:2. If one of the bones of the model is 8.25 centimeters long, how long is the actual bone in a human ear?

14. **TELEVISION**  The ratio of the length of a wide-screen TV to its width is 16:9. Find the width of a wide-screen TV if the length measures 28 inches. Round to the nearest tenth.

15. **CHALLENGE**  Suppose two rectangles are similar with a scale factor of 2. What is the ratio of their areas? Explain.

**Writing in Math**  Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

16. Any two rectangles are similar.

17. Any two squares are similar.

**TEST PRACTICE**

18. Triangle $FGH$ is similar to triangle $RST$.

$\triangle FGH$:
- $FG = 36$ in.
- $GH = 18$ in.
- $FH = 34$ in.

$\triangle RST$:
- $RS = ?$
- $ST = 27$ in.

What is the length of $\overline{TS}$?

A. $13\frac{1}{2}$ inches  
B. $22\frac{2}{3}$ inches  
C. 24 inches  
D. $25\frac{1}{2}$ inches

19. Quadrilateral $ABCD$ is similar to quadrilateral $WXYZ$.

$\square ABCD$:
- $AB = 4$ in.
- $BC = 8$ in.
- $CD = 6$ in.
- $DA = 6$ in.

$\square WXYZ$:
- $WX = ?$
- $XY = 4$ in.
- $YZ = 8$ in.
- $WZ = 6$ in.

If the perimeter of quadrilateral $ABCD$ is 54 units, what is the perimeter of quadrilateral $WXYZ$?

F. 13.5 inches  
G. 24 inches  
H. 27 inches  
J. 36 inches

**Spiral Review**

20. **ROCK CLIMBING**  Grace is working her way up a climbing wall. Every 5 minutes she is able to climb 6 feet, but then loses her footing, slips back 1 foot, and decides to rest for 1 minute. If the rock wall is 30 feet tall, how long will it take her to reach the top? Use the draw a diagram strategy.  

Solve each proportion.  

21. $\frac{5}{4} = \frac{y}{12}$  
22. $\frac{120}{b} = \frac{24}{60}$  
23. $\frac{0.6}{5} = \frac{1.5}{n}$

**GET READY for the Next Lesson**

**PREREQUISITE SKILL**  Graph and connect each pair of ordered pairs.

24. $(-2.5, 1.5), (1.5, -3.5)$  
25. $(-2, -1\frac{1}{2}), (4, 3\frac{1}{2})$  
26. $(-2\frac{1}{3}, 1), (2, 3\frac{2}{3})$