

# 3-1

## Square Roots (pages 116–119)

Numbers that can be written as  $p \cdot p$  where  $p$  is an integer or a rational number, are called **perfect squares**. For example, 9, 25, 0.09,  $\frac{4}{9}$ , and  $\frac{36}{81}$  are perfect squares.

<b>Finding Square Roots</b>	<ul style="list-style-type: none"> <li>When <math>n = r^2</math>, then <math>r</math> is a <b>square root</b> of <math>n</math>.</li> <li>Notice that <math>36 = 6 \cdot 6</math> and <math>36 = (-6) \cdot (-6)</math>, so both 6 and <math>-6</math> are square roots of 36. Sometimes we want only the positive square root.</li> <li>The positive square root of a number is called the <b>principal square root</b>. The symbol <math>\sqrt{\quad}</math>, called a <b>radical sign</b>, is used to indicate the principal square root. <math>\sqrt{36} = 6</math></li> <li>Indicate the negative square root like this. <math>-\sqrt{36} = -6</math></li> </ul>
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### EXAMPLES

**A** Find  $\sqrt{900}$ .

*Ask: what number multiplied by itself gives 900?*

$30 \cdot 30 = 900$ , so  $\sqrt{900} = 30$ .

**B** Find  $-\sqrt{\frac{25}{121}}$ .

*Notice that you are finding the negative square root.*

$-\sqrt{\frac{25}{121}} = -\frac{5}{11}$

### Try These Together

<p>1. Find <math>\sqrt{49}</math>. <i>HINT: Find <math>n</math> if <math>n \cdot n = 49</math>.</i></p>	<p>2. Find <math>-\sqrt{16}</math>. <i>HINT: This root will be a negative integer.</i></p>
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### PRACTICE

**Find each square root.**

3.  $\sqrt{144}$

4.  $\sqrt{\frac{9}{25}}$

5.  $\sqrt{676}$

6.  $-\sqrt{225}$

7.  $-\sqrt{\frac{36}{144}}$

8.  $\sqrt{3.61}$

9.  $\sqrt{\frac{169}{400}}$

10.  $\sqrt{0.81}$

**Solve each equation.**

11.  $x^2 = 64$

12.  $x^2 = 5.76$

13.  $x^2 = \frac{9}{16}$



**14. Standardized Test Practice** You are arranging chairs for the school show.

You have 256 chairs to arrange in a square. How many rows of chairs would you need and how many chairs in each row would you have?

**A** 16; 16

**B** 20; 20

**C** 16; 20

**D** 4; 4

Answers: 1. 7 2. -4 3. 12 4. $\frac{5}{3}$ 5. 26 6. -15 7. $-\frac{2}{1}$ 8. 1.9 9. $\frac{20}{13}$ 10. 0.9 11. 8, -8 12. 2.4, -2.4 13. $\frac{4}{3}$ , $-\frac{4}{3}$ 14. A
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**3-2****Estimating Square Roots** (pages 120–122)

You can estimate the square roots of numbers that are not perfect squares.

**Estimating Square Roots**

To estimate the square root of  $r$ , find perfect squares on each side of  $r$ . Use these to estimate.

**EXAMPLES**

- A** Estimate  $\sqrt{38}$  to the nearest whole number.

Find a perfect square a little less than 38 and one a little more than 38.  $\sqrt{36} < \sqrt{38} < \sqrt{49}$ , so  $6 < \sqrt{38} < 7$ . Since 38 is closer to 36 than 49, the best whole number estimate for  $\sqrt{38}$  is 6.

- B** Estimate  $\sqrt{21.6}$  to the nearest whole number.

Find a perfect square a little less than and a little more than 21.6.  $\sqrt{16} < \sqrt{21.6} < \sqrt{25}$ , so  $4 < \sqrt{21.6} < 5$ . Since 21.6 is closer to 16, the best whole number estimate for  $\sqrt{21.6}$  is 5.

**Try These Together**

1. Estimate  $\sqrt{69}$  to the nearest whole number.

*HINT: 69 is between the perfect squares 64 and 81.*

2. Estimate  $\sqrt{7}$  to the nearest whole number.

*HINT: Find the closest perfect squares on each side of 8.*

**PRACTICE**

**Estimate to the nearest whole number.**

3.  $\sqrt{27}$

4.  $\sqrt{147}$

5.  $\sqrt{120}$

6.  $\sqrt{95}$

7.  $\sqrt{254}$

8.  $\sqrt{54}$

9.  $\sqrt{490}$

10.  $\sqrt{313}$

11.  $\sqrt{1.25}$

12.  $\sqrt{101}$

13.  $\sqrt{399}$

14.  $\sqrt{17.4}$

15. **Sewing** You are covering the top of a square stool with felt. The area of the top is 140 square inches. Estimate the length of one side of the top of the stool.



16. **Standardized Test Practice** How many whole numbers are there whose square roots are greater than 9 but less than 10?

A 10

B 15

C 18

D 22

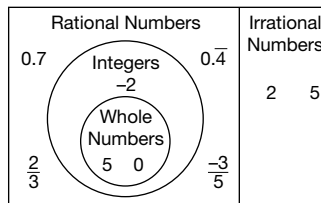
Answers: 1. 8 2. 3 3. 5 4. 12 5. 11 6. 10 7. 16 8. 7 9. 22 10. 18 11. 1 12. 10 13. 20 14. 4 15. 12 in. 16. C

# 3-3

## The Real Number System (pages 125–129)

You have studied whole numbers, integers, and rational numbers. Rational numbers include terminating and repeating decimals as well as the square roots of perfect squares. Numbers that do not terminate or repeat are called **irrational numbers**.

<b>Irrational Numbers</b>	An irrational number is a number that cannot be expressed as $\frac{a}{b}$ , where $a$ and $b$ are integers and $b$ does not equal 0. The square roots of numbers that are not perfect squares are irrational. You can use a calculator to find approximate square roots with numbers such as $\sqrt{11}$ and $\sqrt{27}$ .
<b>Real Numbers</b>	The sets of rational and irrational numbers combine to form the set of <b>real numbers</b> . The graph of all real numbers is the entire number line.



### Try These Together

- Use the letters given below in the Practice exercises to name the set or sets of numbers to which 25 belongs.  
*HINT: You can write 25 as  $\frac{25}{1}$ .*
- Use the letters given below in the Practice exercises to name the set or sets of numbers to which  $\sqrt{47}$  belongs.  
*HINT: 47 is not a perfect square.*

### PRACTICE

Let  $R$  = real numbers,  $Q$  = rational numbers,  $Z$  = integers,  $W$  = whole numbers, and  $I$  = irrational numbers. Name all sets of numbers to which each real number belongs.

- $-\frac{7}{12}$
- $0.272272227 \dots$
- $-\sqrt{16}$

Estimate each square root to the nearest tenth.

- $\sqrt{10}$
- $-\sqrt{31}$
- $\sqrt{77}$
- $\sqrt{124}$



10. **Standardized Test Practice** You are building a fence around your mother's square garden. She has told you that she believes that the garden is about 250 square feet. About how many feet of fence must you purchase in order to enclose the entire garden?

- A** 15 ft                      **B** 16 ft                      **C** 50 ft                      **D** 63 ft

Answers: 1. W, Z, Q, R 2. I, R 3. Q, R 4. I, R 5. Z, Q, R 6. Q, R 7. -5, 6 8. 8.8 9. 11.1 10. D

# 3-4

## The Pythagorean Theorem (pages 132–136)

The longest side of a right triangle is the **hypotenuse**. The sides that form the right angle are the **legs**.

<b>Pythagorean Theorem</b>	In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. $c^2 = a^2 + b^2$	
<b>Converse of Pythagorean Theorem</b>	If the sides of a triangle have lengths $a$ , $b$ , and $c$ units such that $c^2 = a^2 + b^2$ , then the triangle is a right triangle.	

### EXAMPLE

Is a triangle that has sides of 3, 5, and 7 a right triangle?  
 Is  $7^2$  equal to  $3^2 + 5^2$ ? No,  $49 \neq 9 + 25$ , so the sides do not fit the converse of the Pythagorean Theorem. It is not a right triangle.

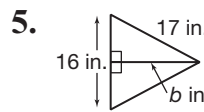
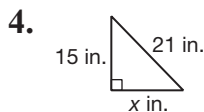
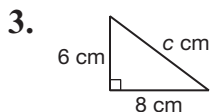
### Try These Together

**Round to the nearest tenth.**

- Find the length of the missing side of the right triangle.  $a$ , 7 m;  $c$ , 11 m  
*HINT: Use  $c^2 = a^2 + b^2$ . Solve for  $b$ .*
- Find the length of the missing side of the right triangle.  $b$ , 24 cm;  $c$ , 37 cm  
*HINT: Use the Pythagorean Theorem.*

### PRACTICE

Find the missing length in each right triangle. Round to the nearest tenth if necessary.



6.  $a$ , 19 yd;  $b$ , 16 yd

7.  $b$ , 67 mm;  $c$ , 69 mm

8.  $a$ , 6.2 m;  $b$ , 8.6 m

Determine whether each triangle with sides of given lengths is a right triangle.

9. 9 in., 12 in., 15 in.

10. 16 ft, 29 ft, 18 ft

11. 9 m, 7 m, 13 m



12. **Standardized Test Practice** The cities of Coldwater, Wayne, and Clinton form a right triangle on the map. The distance from Wayne to Coldwater is 50 miles. The distance from Coldwater to Clinton is 60 miles. Coldwater is due north of Wayne, and Clinton is due east of Coldwater. To the nearest mile, how far is it if you drive directly from Wayne to Clinton?

**A** 55 mi

**B** 67 mi

**C** 78 mi

**D** 110 mi

Answers: 1. 8.5 m 2. 28.2 cm 3. 10 cm 4. 14.7 in. 5. 15 in. 6. 24.8 yd 7. 16.5 mm 8. 10.6 m 9. yes 10. no 11. no 12. C

**3-5****Using the Pythagorean Theorem** (pages 137–140)

You can use the Pythagorean Theorem to find lengths of objects that have rectangular or right-triangular shapes.

**EXAMPLE**

Marcia has a rectangular scarf that measures 36 inches by 48 inches. She folds it along the diagonal to make a right triangle. How long is the hypotenuse?

$$\begin{aligned} 36^2 + 48^2 &= d^2 && \text{Pythagorean Theorem} \\ 1,296 + 2,304 &= d^2 \\ 3,600 &= d^2 \\ 60 &= d \end{aligned}$$

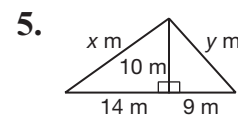
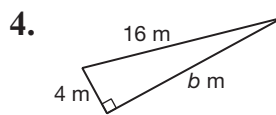
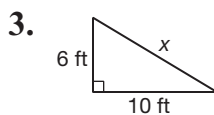
The hypotenuse is 60 inches long.

**Try These Together**

- Determine the length of the second leg of a right triangle that has a hypotenuse of 50 inches and a leg of 40 inches.  
*HINT: Use the Pythagorean Theorem.*
- A table top is 3 feet by 4 feet. How long is its diagonal?  
*HINT: Draw a sketch. What kind of triangle does the diagonal make?*

**PRACTICE**

**Write an equation that can be used to find the length of the missing side of each right triangle. Then solve. Round to the nearest tenth.**



6. **Recreation** A sail on a ship is a right triangle. If one leg measures 30 feet and the other measures 16 ft, find the length of the hypotenuse of the sail.



7. **Standardized Test Practice** A right triangle has one leg that is 18 centimeters and a hypotenuse that is 30 centimeters. Find the length of the third side.

**A** 24 cm

**B** 48 cm

**C** 35 cm

**D** 540 cm

**Answers:** 1. 30 in. 2. 5 ft 3.  $6^2 + 10^2 = x^2$ ;  $x = \sqrt{136} \approx 11.7$  ft 4.  $4^2 + b^2 = 16^2$ ;  $b = \sqrt{240} \approx 15.5$  m 5.  $14^2 + 10^2 = x^2$ ;  $x = \sqrt{296} \approx 17.2$  m;  $9^2 + 10^2 = y^2$ ;  $y = \sqrt{181} \approx 13.5$  m 6. 34 ft 7. A

# 3-6

## Distance on the Coordinate Plane (pages 142–145)

You can use what you know about right triangles to find the distance between two points on a coordinate grid.

<b>Finding Distance on the Coordinate Plane</b>	To find the distance between two points on the coordinate plane, draw the segment that joins the points. Then make that segment the hypotenuse of a right triangle. Use the Pythagorean Theorem to find the length of the hypotenuse, which is the distance between the two points.
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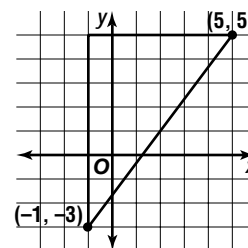
### EXAMPLE

Find the distance between the points  $(5, 5)$  and  $(-1, -3)$ .

First draw the segment that joins these two points. Then draw segments so that this segment is the hypotenuse of a right triangle. Count squares to find the lengths of the legs, 6 and 8. Since 6 and 8 are the first two parts of a Pythagorean triple, you know that the length of the hypotenuse is 10.

Check: Does  $6^2 + 8^2 = 10^2$ ? Yes, because  $36 + 64 = 100$ .

The distance between the two points is 10 units.



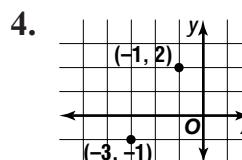
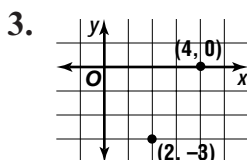
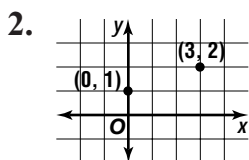
### Try This Together

- Find the distance between  $(7, 3)$  and  $(2, -1)$ . Round to the nearest tenth.

*HINT: Graph the points and then draw segments down from  $(7, 3)$  and to the right from  $(2, -1)$ .*

### PRACTICE

Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth.



Find the distance between the points. Round to the nearest tenth.

5.  $(-3, 3), (2, 0)$       6.  $(4, 4), (-1, -1)$       7.  $(0, 0), (-6, 2)$       8.  $(0, -3), (4, 3)$

9. **Geometry** A right triangle on the coordinate plane has vertices  $A(3, 2)$ ,  $B(-1, -2)$ , and  $C(3, -2)$ . Find the length of the hypotenuse.



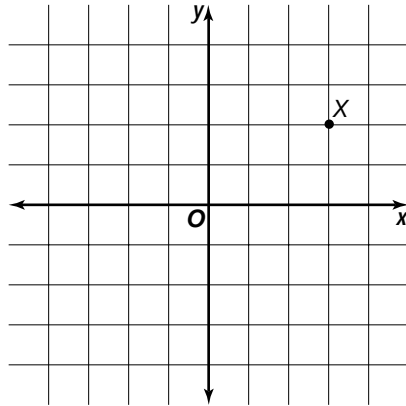
10. **Standardized Test Practice** Find the distance between  $A(8, 4)$  and  $B(0, -2)$ .

- A** 48 units                      **B** 100 units                      **C** 10 units                      **D** 64 units

**Answers:** 1. 6.4 units    2. 3.2 units    3. 3.6 units    4. 3.6 units    5. 5.8 units    6. 7.1 units    7. 6.3 units    8. 7.2 units    9. 5.7 units    10. C

**3****Chapter 3 Review****Coordinate Treasure Hunt**

Starting at point  $X$  on the coordinate plane below, follow the directions to find the location of a hidden treasure. Record your location at each point.



1. Draw a triangle with vertices at  $X$ ,  $Y(3, -1)$  and  $Z(0, -1)$ . What is the measure of angle  $XZY$ ?
2. To the nearest tenth, what is the length of the hypotenuse of this triangle?
3. From point  $Z$  draw a segment to  $W(0, 3)$  and from  $W$ , draw a segment to  $R(-2, 3)$ . To the nearest tenth, what is the measure of  $\overline{RZ}$ ?
4. From point  $R$ , move 6 units south (or down). Where are you now?
5. From there, move 3 units east (or right) to find the treasure. What are the coordinates of the hidden treasure?
6. To the nearest tenth, how far is the treasure from point  $X$ ?

Answers are located on page 108.