

4-1

Ratios and Rates (pages 156–159)

Ratio	A ratio compares two numbers by division. $\frac{27}{100}$, 27 out of 100, 27 to 100, 27:100
Rate	A rate is a special kind of ratio. A rate compares two quantities with different units, such as miles to the gallon or cents per pound.
Unit Rate	When a rate is simplified so that it has a denominator of 1, it is called a unit rate .

EXAMPLES

A Express 12 winners for every 90 people who enter as a rate in simplest form.

Write a fraction for the rate: $\frac{12}{90}$.

Divide numerator and denominator by the GCF to simplify. The GCF of 12 and 90 is 6.

$\frac{2}{15}$ is the rate in simplest form.

B Express the rate \$6 for 3 pounds as a unit rate.

Write a rate: $\frac{\$6}{3 \text{ pounds}}$.

Divide numerator and denominator by 3 to get a denominator that is 1 unit.

The unit rate is \$2 per pound.

Try These Together

1. Express 16 out of 32 in simplest form.

HINT: Write a fraction and simplify.

2. Express 6 wins in 10 games in simplest form.

HINT: Write a fraction and simplify.

PRACTICE

Express each ratio or rate in simplest form.

3. 3 to 15

4. 3 boys: 24 girls

5. 13 meters per second

6. 56 dogs to 48 cats

7. 4 feet: 16 feet

8. 12 books for 4 students

Express each rate as a unit rate.

9. \$18.00 for 3 pounds

10. \$19.50 for 15 gallons

11. \$1.68 for 8 ounces

12. \$2.00 for 10 minutes

13. 8 feet in 2 seconds

14. 25 magazines in 5 days

15. **Sports** Gloribel ran the 400-meter dash in 80 seconds. How many meters did she run per second?



16. **Standardized Test Practice** Suppose that a bottle of peppercorn ranch salad dressing costs \$2.65 at the grocery store. If there are 20 ounces in the bottle, what is the price of the salad dressing per ounce? Round to the nearest cent.

A \$0.14

B \$0.12

C \$0.15

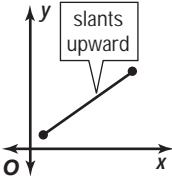
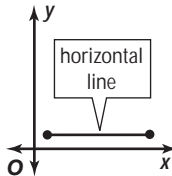
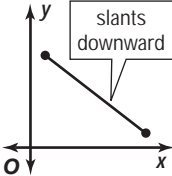
D \$0.13

Answers: 1. $\frac{2}{3}$ 2. $\frac{3}{5}$ 3. $\frac{5}{1}$ 4. $\frac{8}{1}$ 5. $\frac{13}{1}$ 6. $\frac{6}{7}$ 7. $\frac{4}{1}$ 8. $\frac{1}{3}$ 9. \$6.00 per pound 10. \$1.30 per gallon 11. \$0.21 per ounce 12. \$0.20 per minute 13. 4 feet per second 14. 5 magazines per day 15. 5 16. D

4-2

Rate of Change (pages 160–164)

A **rate of change** is a rate that describes how one quantity changes in relation to another. To find the rate of change, divide the difference in the y -coordinates by the difference in the x -coordinates. The rate of change between (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$. Rates of change can be positive, negative, or zero.

Rate of Change	positive	zero	negative
Real-Life Meaning	increase	no change	decrease
Graph			

EXAMPLE

Find the rate of change between 1990 and 2000.

$$\frac{\text{change in population}}{\text{change in year}} = \frac{(1,293,953 - 1,006,749) \text{ people}}{(2000 - 1990) \text{ years}}$$

$$= \frac{287,204 \text{ people}}{10 \text{ years}} = \frac{28,720.4 \text{ people}}{1 \text{ year}}$$

The population of Idaho has grown an average of 28,720.4 people per year.

Population of Idaho	Year
588,637	1950
667,191	1960
713,015	1970
944,127	1980
1,006,749	1990
1,293,953	2000

The World Almanac, 2002, p. 377

PRACTICE

For Exercises 1–4, use the table at the right. The table shows the number of patrons at the local swimming pool throughout the day.

- Find the rate of change from 12 P.M. to 1 P.M.
- Find the rate of change from 11 A.M. to 2 P.M.
- Was the rate of change between 1 P.M. and 2 P.M. positive, negative, or zero?
- During which time period was the rate of change in patrons negative?

Time	Number of Patrons at the Swimming Pool
11 A.M.	12
12 P.M.	23
1 P.M.	25
2 P.M.	25
3 P.M.	13



- Standardized Test Practice** At West High School the T-shirt sales for the pep club totaled 135 in 1999. In 2002, they totaled 162. If this rate of change were to continue, what would be the total T-shirt sales in 2003?

A 171 T-shirts **B** 153 T-shirts **C** 162 T-shirts **D** 135 T-shirts

Answers: 1. 2 people/hour 2. ≈ 4.3 people/hour 3. zero 4. between 2 P.M. and 3 P.M. 5. A

4-3

Slope (pages 166–169)

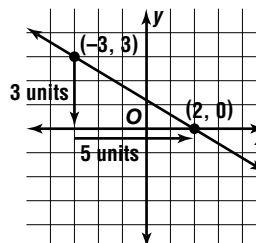
The rate of change between any two points on a line is always the same. This constant rate of change is called the slope of the line. **Slope** is the ratio of the **rise**, or vertical change, to the **run**, or horizontal change.

EXAMPLE

Find the slope of the line.

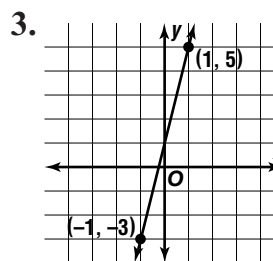
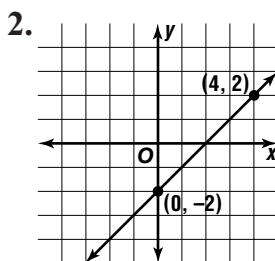
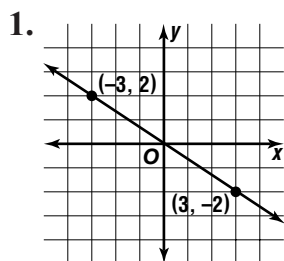
Choose two points on the line. The vertical change is down 3 units, or -3 , while the horizontal change is right 5 units, or $+5$.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-3}{5}$$



PRACTICE

Find the slope of each line.



The points given in each table lie on a line. Find the slope of the line. Then graph the line.

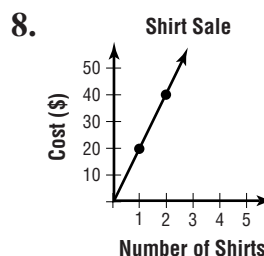
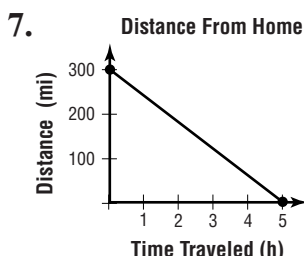
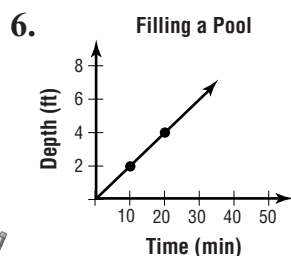
4.

x	-1	0	1	2
y	5	3	1	-1

5.

x	-8	-4	0	4
y	-3	0	3	6

Find the slope of each line and interpret its meaning as a rate of change.



9. **Standardized Test Practice** There are two ramps that enter the school.

The first rises 2 feet for every 16-foot run. The second ramp rises 1 foot for every 7-foot run. Which statement is true?

- A** The first ramp is steeper than the second.
- B** Both ramps have the same steepness.
- C** The second ramp is steeper than the first.
- D** This cannot be determined from the information given.

Answers: 1. $-\frac{3}{5}$ 2. 1 3. 4 4-5. See Answer Key for graphs. 4. -2 5. $\frac{4}{3}$ 6. $\frac{5}{1}$ 7. The pool fills at a rate of $\frac{5}{1}$ foot per minute. 8. 20; Each shirt costs \$20. 9. C

4-4**Solving Proportions** (pages 170–173)

You can use two equal ratios to write a proportion.

Solving a Proportion	<p>A proportion is an equation stating that two ratios are equivalent.</p> $\frac{a}{b} = \frac{c}{d}, b \neq 0, \text{ and } d \neq 0$ <p>The cross products of a proportion are equal. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.</p>
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EXAMPLES

- A** Determine whether the ratios $\frac{2}{3}$ and $\frac{3}{4}$ form a proportion.

Are the cross products for $\frac{2}{3}$ and $\frac{3}{4}$ equal?

The cross products are 2×4 and 3×3 . $8 \neq 9$.

Since the cross products are not equal, $\frac{2}{3} \neq \frac{3}{4}$, the ratios do not form a proportion.

- B** Solve $\frac{4}{5} = \frac{12}{c}$.

Find the cross products.

$$4 \times c = 5 \times 12$$

$$4c = 60$$

$$\frac{4c}{4} = \frac{60}{4}$$

$$c = 15$$

Divide each side by 4.

Try These Together

1. Determine whether $\frac{3}{5}$ and $\frac{2}{4}$ form a proportion.

HINT: Find the cross products.

2. Determine whether $\frac{6}{8}$ and $\frac{3}{4}$ form a proportion.

HINT: See if the cross products are equal.

PRACTICE

Determine whether each pair of ratios form a proportion.

3. $\frac{10}{20}, \frac{6}{12}$

4. $\frac{3}{8}, \frac{1}{5}$

5. $\frac{2}{6}, \frac{8}{24}$

6. $\frac{5}{25}, \frac{1}{5}$

7. $\frac{6}{15}, \frac{2}{5}$

8. $\frac{9}{27}, \frac{5}{12}$

Solve each proportion.

9. $\frac{2}{5} = \frac{x}{20}$

10. $\frac{3}{n} = \frac{4}{8}$

11. $\frac{3}{p} = \frac{6}{16}$

12. $\frac{6}{10} = \frac{3}{r}$

13. $\frac{a}{5} = \frac{15}{25}$

14. $\frac{y}{7} = \frac{9}{21}$

15. $\frac{6}{4} = \frac{t}{8}$

16. $\frac{3}{9} = \frac{9}{k}$

- 17. Manufacturing** A company manufactures two different types of school desks. One is a desk with the chair attached and the other is a small desk with a separate chair. One out of every 3 desks they manufacture has the chair separate. If they manufactured 90 desks, how many would have the chairs separate?



- 18. Standardized Test Practice** If a car can travel 60 miles in 1 hour, how far can it travel in 5 hours?

A 300 mi

B 1,100 mi

C 600 mi

D 550 mi

Answers: 1. no 2. yes 3. yes 4. no 5. yes 6. yes 7. yes 8. no 9. 8 10. 6 11. 8 12. 5 13. 3 14. 3 15. 12 16. 27

4-5

Similar Polygons (pages 178–182)

A **polygon** is a simple closed figure in a plane formed by three or more line segments. A **quadrilateral** is a polygon with four sides. A **pentagon** is a polygon with five sides.

Similar Polygons

Two polygons are **similar** if their corresponding angles are congruent, and their corresponding sides are proportional.

EXAMPLE

In the figure at the right, $\triangle ABC \sim \triangle DEF$. Find the length of side \overline{DE} .

\overline{AB} corresponds to \overline{DE} and \overline{BC} corresponds to \overline{EF} . So you can write a proportion.

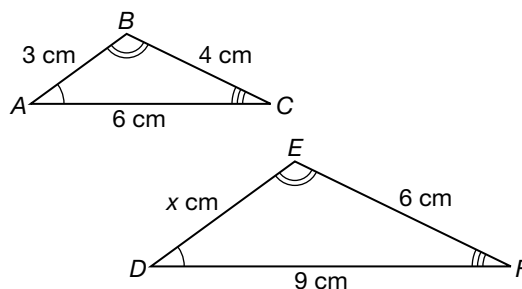
$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{3}{x} = \frac{4}{6} \quad AB = 3, DE = x, BC = 4, EF = 6$$

$$18 = 4x \quad \text{Find the cross products.}$$

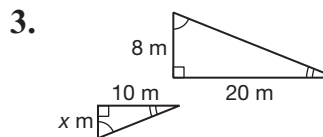
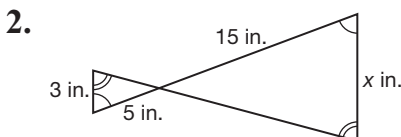
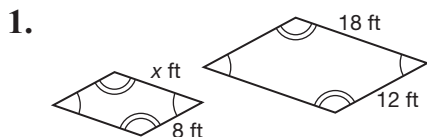
$$4.5 = x \quad \text{Solve for } x.$$

The length of \overline{DE} is 4.5 centimeters.



PRACTICE

Each pair of polygons is similar. Write a proportion to find each missing measure. Then solve.



4. **Hobbies** Sean wants to enlarge a 4-inch by 6-inch photo so the shortest side is 6 inches. How long will the longest side be?



5. **Standardized Test Practice** $\triangle ABC$ is similar to $\triangle DEF$. If $AB = 2$, $BC = 5$, and $DE = 26$, then EF is equal to what?

A $2\frac{4}{5}$

B $10\frac{2}{5}$

C $20\frac{4}{5}$

D 65

Answers: 1–3. Sample proportions are given. 1. $\frac{12}{8} = \frac{18}{x}$; 2. $\frac{18}{x} = \frac{12}{5}$; 3. $\frac{8}{10} = \frac{15}{x}$; 4. $\frac{20}{4} = \frac{6}{x}$; 5. D

4-6

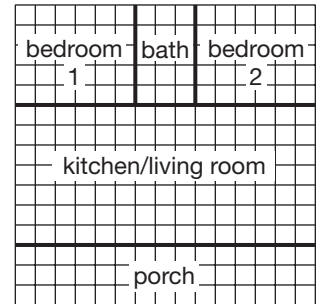
Scale Drawings and Models (pages 184–187)

A **scale drawing** or **scale model** is used to represent an object that is too large or too small to be drawn or built at actual size.

Using Scale Drawings	The scale of a drawing or model is determined by the ratio of a given length on the drawing or model to its corresponding actual length.
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EXAMPLE

The figure at the right is a scale drawing of a cabin plan. In the drawing, the side of each square represents 20 inches. Find the length and width of bedroom 2.



Count the squares in the scale drawing. Bedroom 2 is 6 squares long and 5 squares wide. Use the scale and your counts to write proportions.

$$\frac{1 \text{ square}}{20 \text{ in.}} = \frac{6 \text{ squares}}{x \text{ in.}} \qquad \frac{1 \text{ square}}{20 \text{ in.}} = \frac{5 \text{ squares}}{y \text{ in.}}$$

$$1 \cdot x = 20 \cdot 6 \qquad 1 \cdot y = 20 \cdot 5$$

$$x = 120 \qquad y = 100$$

The length of bedroom 2 is 120 inches, and the width is 100 inches.

Try These Together

- Use the figure and scale in the Example to find the length and width of the kitchen/living room.
HINT: Write proportions.
- Use the figure and scale in the Example to find the length and width of the porch.
HINT: The length is the same as the kitchen/living room.

PRACTICE

- Find the length and width of the bath in the Example.
- On a map, the scale is 1 inch = 250 miles. Find the actual distance for each map distance.

	From	To	Map Distance
a.	Minneapolis, Minnesota	San Diego, California	about 8 inches
b.	San Diego, California	Portland, Oregon	about $4\frac{1}{4}$ inches
c.	Portland, Oregon	Minneapolis, Minnesota	about 7 inches



5. **Standardized Test Practice** Find the dimensions of the cabin (including the porch) in the Example.

- A** 150 in. by 150 in. **B** 112 in. by 112 in.
C 300 in. by 280 in. **D** 300 in. by 300 in.

Answers: 1. 300 in. by 140 in. 2. 300 in. by 60 in. 3. 60 in. by 100 in. 4a. about 2,000 miles 4b. about 1,062.5 miles 4c. about 1,750 miles 5. D
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4-7

Indirect Measurements (pages 188–191)

Using proportions to find a measurement is called **indirect measurement**.

Using Indirect Measurement	Use the corresponding parts of similar triangles to write a proportion. Solve the proportion to find the missing measurement.
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EXAMPLE

George is $5\frac{1}{2}$ feet tall. His shadow is 22 inches long at the same time that a tree has a shadow that is 120 inches long. How many feet tall is the tree?

$$\frac{5.5 \text{ feet}}{22 \text{ inches}} = \frac{t \text{ feet}}{120 \text{ inches}} \quad \text{Write a proportion.}$$

$$5.5(120) = 22t \quad \text{Find the cross products.}$$

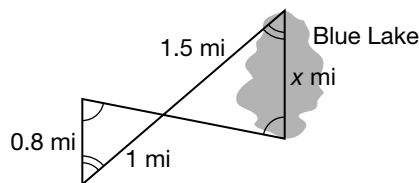
$$30 = t \quad \text{Solve for } t.$$

The tree is 30 feet tall.

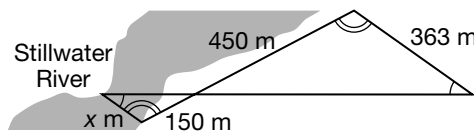
PRACTICE

In Exercises 1–3, the triangles are similar. Write a proportion and solve the problem.

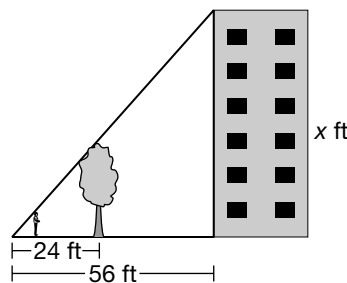
1. Find the distance across Blue Lake.



2. The city of Hutchinson plans to build a bridge over the narrowest part of Stillwater River. Find the distance across this part of the river.



3. When Peter stands in front of a 27-foot tree in front of his apartment building he can barely see the very top of the building over the tree. How tall is his apartment building?



4. **Standardized Test Practice** $\triangle ABC \sim \triangle XYZ$. $AB = 45$ m, $BC = 15$ m, and $XY = 24$ m. How long is YZ ?

A $2\frac{2}{3}$ m

B $7\frac{2}{3}$ m

C 8 m

D 72 m

Answers: 1–3. Sample proportions are given. 1. $\frac{0.8}{x} = \frac{1.5}{1.2}$; $1.2 \text{ mi} = \frac{1.5}{1.5} \cdot x$; $x = 1.2 \text{ mi}$ 2. $\frac{363}{x} = \frac{450}{150}$; $121 \text{ m} = \frac{450}{150} \cdot x$; $x = 121 \text{ m}$ 3. $\frac{27}{x} = \frac{24}{63}$; $63 \text{ ft} = \frac{24}{27} \cdot x$ 4. C

4-8

Dilations (pages 194–197)

The image produced by enlarging or reducing a figure is called a **dilation**.

Working with Dilations	Since the dilated image has the same shape as the original, the two images are similar. The ratio of the dilated image to the original is called the scale factor .
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EXAMPLE

A triangle has vertices $M(2, -2)$, $N(6, -2)$, and $P(2, 4)$. Find the coordinates of $\triangle MNP$ after a dilation with a scale factor of $\frac{5}{2}$.

Multiply each coordinate in each ordered pair by $\frac{5}{2}$.

$$M(2, -2) \rightarrow \left(2 \cdot \frac{5}{2}, -2 \cdot \frac{5}{2}\right) \rightarrow M'(5, -5)$$

$$N(6, -2) \rightarrow \left(6 \cdot \frac{5}{2}, -2 \cdot \frac{5}{2}\right) \rightarrow N'(15, -5)$$

$$P(2, 4) \rightarrow \left(2 \cdot \frac{5}{2}, 4 \cdot \frac{5}{2}\right) \rightarrow P'(5, 10)$$

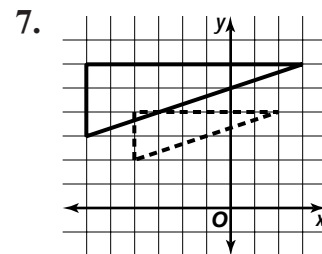
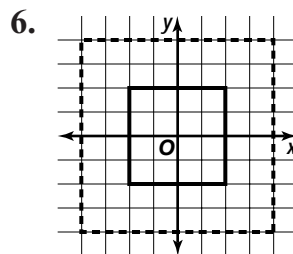
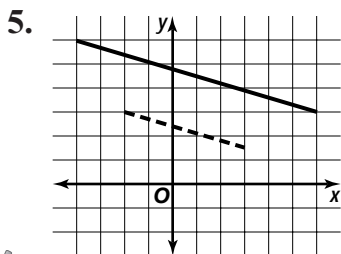
PRACTICE

- Find the coordinates of the image of point $C(12, 4)$ after a dilation with a scale factor of $\frac{2}{3}$.

Triangle KLM has vertices $K(-5, 15)$, $L(-5, -10)$, and $M(15, 20)$. Find the coordinates of its vertices after a dilation with each given scale factor.

- 3
- $\frac{1}{5}$
- $\frac{3}{5}$

In each figure, the dashed-lined figure is a dilation of the solid-lined figure. Find each scale factor.



- Standardized Test Practice** What are the coordinates of the image of point $Q(3, 8)$ after a dilation with a scale factor of $\frac{1}{4}$?

- A** $Q'\left(\frac{3}{4}, 2\right)$ **B** $Q'(12, 32)$ **C** $Q'(3, 2)$ **D** $Q'\left(\frac{4}{3}, \frac{1}{2}\right)$

4. $K'(-3, 9)$, $L'(-3, -9)$, $M'(9, 12)$ 5. $\frac{2}{1}$ 6. 2 7. $\frac{3}{2}$ 8. A Answers: 1. $C'(8, 2\frac{3}{2})$ 2. $K'(-15, 45)$, $L'(-15, -45)$, $M'(45, -30)$, $M'(45, 60)$ 3. $K'(-1, 3)$, $L'(-1, -1)$, $M'(3, 4)$

4

Chapter 4 Review

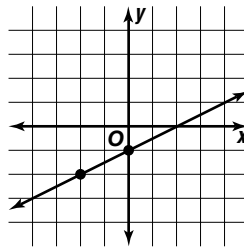
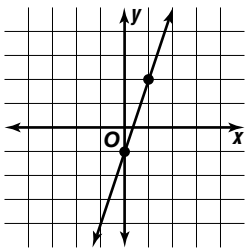
Vocabulary Time

Solve each problem. Find the letter from the list at the bottom of the page that corresponds to your numerical answer. Place the letter in the blank at the right. When you are finished you will have spelled a vocabulary word from the chapter.

1. Express the ratio in simplest form: 9 Aspens to 12 trees. 1. _____

2. Express the rate as a unit rate: \$12 for 24 donuts. 2. _____

3. Find the slope of the line. 4. Find the slope of the line. 3. _____



4. _____

5. Write a proportion that could be used to solve for m . Then solve.
4 miles run in 30 minutes, 6 miles run in m minutes. 5. _____

6. Segment $A'B'$ is a dilation of segment AB . The endpoints of each segment are $A(-2, \frac{1}{2})$, $B(1\frac{1}{2}, 3)$, $A'(-4, -1)$, and $B'(3, 6)$. Find the scale factor of the dilation. 6. _____

7. Corey is 5 feet 6 inches tall. He stands next to a tree that casts a shadow of 37 feet 6 inches. If Corey's shadow is 8 feet 3 inches, how tall is the tree in feet? 7. _____

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	11	4	9	$\frac{3}{5}$	0	6	15	$\frac{1}{2}$	$\frac{4}{3}$	41	45	3	$\frac{7}{3}$	12	30	18	25	$\frac{3}{4}$	7	$\frac{2}{3}$	5	10	1	27	8

Answer is located on page 108.